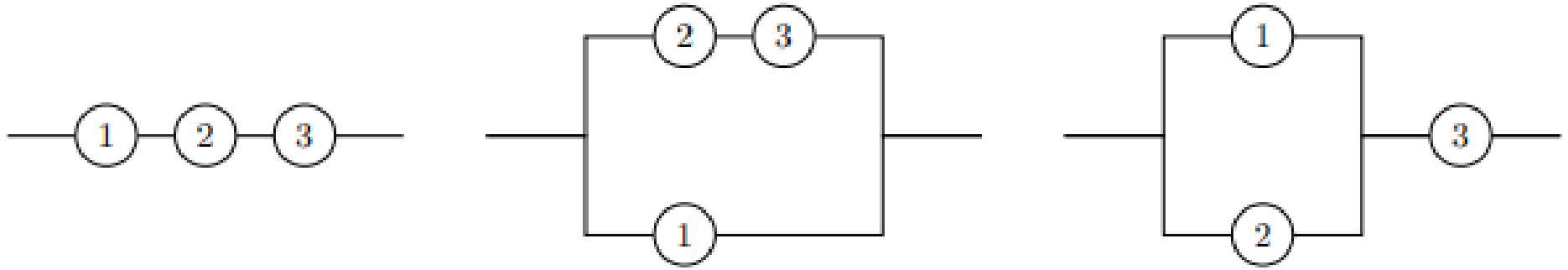
80. (**). The diagrams below represent systems with components that fail independently. For each of them: (i) find the probability that the system works, as a function of p_1, p_2, and p_3, (ii) find the probability that the system works in case $p_1 = p_2 = p_3 = 0.7$, (iii) find the conditional probability that component 1 works, given that the system works, in case $p_1 = p_2 = p_3 = 0.7$.



(a) (b) (c)

82. (**). The colour of a certain kind of mouse is determined by the inheritance of allele pairs of a gene, which are denoted BB, Bb and bb. B is dominant (BB or Bb give a black mouse, while bb gives a brown mouse). When mating, each parent passes one of its allele pair (each choice being equally likely) independently for each offspring.

In a certain experiment, two mice of type Bb are mated to produce an offspring, Mickey.

(a) Find the probability that Mickey is black.

For parts (b)–(d) suppose that Mickey is indeed black.

(b) What is the probability that Mickey is type BB?

(c) Mickey is mated with a brown mouse. They have three offspring. Find the probability that all the offspring are black.

(d) Suppose that all three offspring of part (c) are indeed black. What is the probability now that Mickey is of type BB?

Finally, suppose that in the original experiment the genotypes of Mickey's parents are unknown. However, each of the two mice, independently, has probability 1/3 of genotype BB, and probability 2/3 of genotype Bb.

(e) Repeat parts (a), (b), (c) and (d) using this information.

88. (**). A bag contains m red marbles and n blue marbles. You randomly take r marbles from the bag, without replacement, with $r \leq m+n$. Let X denote the number of red marbles you end up with. Find the probability mass function $p(x)$ for all $x \in \mathbb{N}$.

90. (*). Suppose X is a discrete random variable with possible values 1, 2, and 3, and with probability mass function $p(x) = cx^2$ for $x \in \{1, 2, 3\}$. Calculate:

(a) the value of the constant c,

(b) the value of $\mathbb{P}(X \geq 2)$, and

(c) the value of $\mathbb{P}(X \in \{1, 3\})$.

92. (*). Suppose $X \sim \text{Bin}(n, p)$. Calculate $p(x)$ when

(i) $n = 7$, $p = 0.3$ for $x = 0, 1, 2$;

(ii) $n = 10$, $p = 0.95$ for $x = 9, 10$;

(iii) $n = 10$, $p = 0.05$ for $x = 1, 0$.

Use these to find $\mathbb{P}(X > 2)$ in part (i), $\mathbb{P}(X \leq 8)$ in part (ii), and $\mathbb{P}(X \geq 2)$ in part (iii).