

Topology (Math 3281)

Homework Problem Set 2

24.10.14

This set of homeworks will be collected in the lecture on 07.11.14.

1. Decide which of the following subsets of $\wp(X)$, where $X = \{1, 2, 4\}$, defines a topology. For those which are topologies, decide whether they are homeomorphic. Write down the homeomorphism for the homeomorphic ones.
 - $\tau_1 = \{\emptyset, \{1\}, \{1, 2\}, \{1, 2, 4\}\}$.
 - $\tau_2 = \{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{1, 2, 4\}\}$.
 - $\tau_3 = \{\emptyset, \{1\}, \{4\}, \{1, 2, 4\}\}$.
 - $\tau_4 = \{\emptyset, \{2\}, \{4\}, \{2, 4\}, \{1, 2, 4\}\}$.
 - $\tau_5 = \{\emptyset, \{2, 4\}, \{1, 2, 4\}\}$.
 - $\tau_6 = \{\emptyset, \{2\}, \{1, 2, 4\}\}$.
 - $\tau_7 = \{\emptyset, \{1\}, \{2\}, \{2, 4\}, \{1, 2, 4\}\}$.
2. Decide which of the following subsets of \mathbb{R}^n is open, and which ones are closed. Justify your statements for each set.
 - $X_1 = \{(x, y, z) \in \mathbb{R}^3 \mid x > y > z\}$.
 - $X_2 = \{(x, y, z, w) \in \mathbb{R}^4 \mid x^4 + y^3 + z^2 = w\}$.
 - $X_3 = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1 \geq x_2^2\}$.
 - $X_4 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid x_1 < x_2 + x_3^3, x_3 \leq 5\}$.
3. Let Y be a subspace of X (with the subspace topology). If A is closed in Y , and if Y is closed in X , show that A is closed in X .
4. Find all the limit points of the following subsets of the real line.
 - (a) $A = \{\frac{1}{n} + \frac{1}{m} \mid n, m \in \mathbb{Z}, n, m \geq 1\}$.
 - (b) $B = \{\frac{1}{n} \sin n \mid n \in \mathbb{Z}, n \geq 1\}$.