Topology (Math 3281)

Problem Class 2 10.11.14

This set of problems will be discussed in the Problem Class on 10.11.14, along with old homework problems.

1. Verify
   (a) $\overline{A \cup B} = \overline{A} \cup \overline{B}$.
   (b) $\overline{A \cap B} \subset \overline{A} \cap \overline{B}$.
   (c) $\overline{\overline{A}} = \overline{A}$.
   (d) Find two subsets $A, B \subset \mathbb{R}$ such that strict inclusion holds in (b).
2. Verify
   (a) $A^\circ \cup B^\circ \subset (A \cup B)^\circ$.
   (b) $A^\circ \cap B^\circ = (A \cap B)^\circ$.
   (c) $(A^\circ)^\circ = A^\circ$.
   (d) Find two subsets $A, B \subset \mathbb{R}$ such that strict inclusion holds in (a).
3. If $X \times Y$ has the product topology, and $A \subset X$, $B \subset Y$, show that
$$\overline{A \times B} \quad = \quad \overline{A} \times \overline{B}.$$
4. Let (M, d) and (M', d') be metric spaces.
   (a) Show that
$$d_P((x, x'), (y, y')) \quad = \quad d(x, y) + d'(x', y')$$
   for $(x, x'), (y, y') \in M \times M'$ defines a metric on $M \times M'$.
   (b) Show that the topology induced by d_P is the same as the product topology coming from d and d'.