Topology (Math 3281)

Problem Class 3 24.11.14

This set of problems will be discussed in the Problem Class on 24.11.14, along with old homework problems.

1. Let $n \geq 2$, $i, j \leq n$ with $i \neq j$, $\lambda, \mu \in \mathbb{R}$ and $A_{ij}(\lambda)$ the $n \times n$ matrix which differs from the identity matrix only in the (i, j)-th entry, where the entry of $A_{ij}(\lambda)$ is λ. Also, for $\mu \neq 0$ let $M_i(\mu)$ be the $n \times n$ matrix which differs from the identity matrix only in the (i, i)-th entry, where the entry of $M_i(\mu)$ is μ.

   (a) Show that there is a path in $GL_n(\mathbb{R})$ from $A_{ij}(\lambda)$ to the identity matrix I_n.

   (b) For $\mu > 0$, show that there is a path in $GL_n(\mathbb{R})$ from $M_i(\mu)$ to the identity matrix I_n.

   (c) Let $GL_n^+(\mathbb{R}) = \{A \in GL_n(\mathbb{R}) \mid \det A > 0\}$. Show that $GL_n^+(\mathbb{R})$ is path connected.

   (d) Show that $GL_n(\mathbb{C})$ is path connected.

2. Let X be a topological space and $(A_i)_{i \in I}$ a collection of subsets such that each A_i is path connected, and for $i, j \in I$ the set $A_i \cap A_j$ is path connected.

   (a) Show that $\bigcup_{i \in I} A_i$ is path connected.

   (b) Show that there is a concept of path components in X such that a path component is a maximal path connected subset of X, and such that each point in X is contained in a unique path component. Do path components have to be closed?