Topology (Math 3281)

Problem Class 4 08.12.14

This set of problems will be discussed in the Problem Class on 08.12.14, along with old homework problems.

1. Find an action of $\mathbb{Z}_2$ on the torus $S^1 \times S^1$ such that the orbit space is homeomorphic to $S^1 \times [0, 1]$.

2. Let the topological group G act on the topological space X. Denote the quotient map by $\pi : X \to X/G$.

   (a) Let $U \subset X$ be open. Show that $\pi^{-1}(\pi(U))$ is the union of the sets $gU = \{g \cdot u \in X \mid u \in U\}$, where $g \in G$. Deduce that π maps open sets to open sets.

   (b) Give an example to show that π does not have to send closed sets to closed sets.

3. Let $U(n) = \{A \in GL_n(\mathbb{C}) \mid AA^* = I\}$, where $A^* = (a^*_{ij})$ is the matrix whose entries satisfy $a^*_{ij} = \bar{a}_{ji}$, where $A = (a_{ij})$, and $\bar{a}$ is the complex conjugate of $a \in \mathbb{C}$. Show that $U(n)$ is a compact topological group.