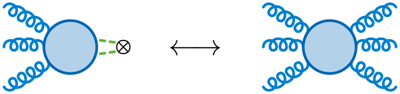
An Antipodal Amplitude/Form Factor Duality

Andrew McLeod

Polylogarithms, Cluster Algebras, and Scattering Amplitudes
September 12, 2023

arXiv:2112.06243 and arXiv:2204.11901 with L. Dixon, Ö. Gürdoğan, and M. Wilhelm



Outline

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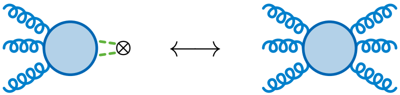
   ◦ Relates three-particle form factors to six-particle amplitudes



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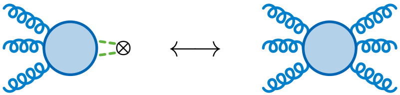

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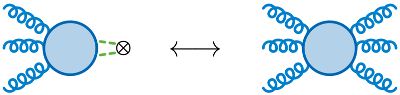

direct connection
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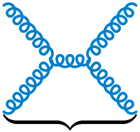
   - Evidence for this duality **through seven loops**
   - **Extends to a self-duality for four particle form factors**

From Feynman Diagrams. . .

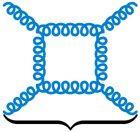
Feynman diagrams provide an intuitive picture for calculating scattering amplitudes and related quantities perturbatively

$\mathcal{A}_{2\to 2} =$



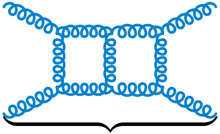
tree level

$+$



one loop

$+$



two loops

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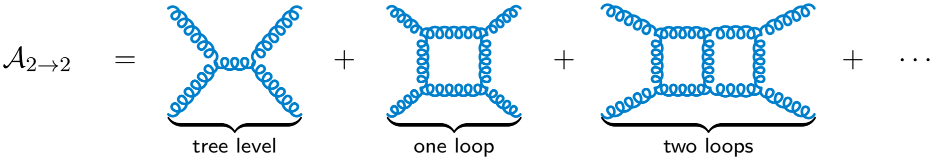

Two sources of difficulty arise when using Feynman diagrams

- number of diagrams grows exponentially

$gg \to$??	gg	ggg	$gggg$	$ggggg$	$gggggg$
# tree diagrams	4	25	220	2485	34300

[Mangano, Parke (1990)]

- at loop level, each diagram becomes a complicated integral over loop momenta

. . . to Surprisingly Simple Expressions

Despite this computational complexity, **scattering amplitudes exhibit striking simplicity**

- At tree level, the n-particle maximum-helicity-violating (MHV) gluon amplitude recombines to

$$\left|\mathcal{A}_n(p_1^-, p_2^-, p_3^+, \ldots, p_n^+)\right|^2 \propto \sum_{\sigma \in S_n} \frac{(p_1 \cdot p_2)^4}{(p_{\sigma_1} \cdot p_{\sigma_2})(p_{\sigma_2} \cdot p_{\sigma_3}) \cdots (p_{\sigma_n} \cdot p_{\sigma_1})}$$

[Parke, Taylor (1986)]

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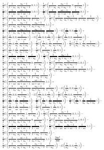
- Similar simplifications occur at loop level

The **two-loop six-particle MHV amplitude** in planar $\mathcal{N} = 4$ supersymmetric Yang-Mills theory

$+ \cdots +$



$\Rightarrow$

$$R_6^{(2)}(u_1, u_2, u_3) = \sum_{i=1}^{3} \left(L_4(x_i^+, x_i^-) - \frac{1}{2} \mathrm{Li}_4(1 - 1/u_i) \right) - \frac{1}{8} \left(\sum_{i=1}^{3} \mathrm{Li}_2(1 - 1/u_i) \right)^2 + \frac{1}{24} J^4 + \frac{\pi^2}{12} J^2 + \frac{\pi^4}{72}. \quad (3)$$

was first computed as a 17 page expression and later simplified to a two-line expression

[Del Duca, Duhr, Smirnov (2009)] [Goncharov, Spradlin, Vergu, Volovich (2010)]

Bootstrap Methods

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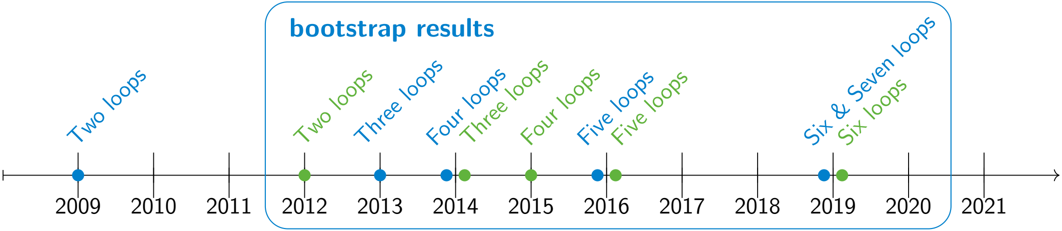


MHV [Del Duca, Duhr, Smirnov (2009)] [Dixon, Drummond, Henn (2011)] [Dixon, Drummond, von Hippel, Pennington (2013)] [Dixon, Drummond, Duhr, Pennington (2014)] [Caron-Huot, Dixon, AJM, von Hippel (2016)] [Caron-Huot, Dixon, Dulat, von Hippel, AJM, Papathanasiou (2019)]

NHMV [Dixon, Drummond, Henn (2012)] [Dixon, von Hippel (2014)] [Dixon, von Hippel, AJM (2015)] [Caron-Huot, Dixon, AJM, von Hippel (2016)] [Caron-Huot, Dixon, Dulat, von Hippel, AJM, Papathanasiou (2019)]

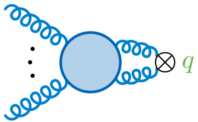
- these bootstrap methods bypass Feynman diagrams altogether, and just try to directly construct the function that has all the right properties to be the amplitude

Form Factors

More recently, bootstap techniques have been used to compute form factors

- Form factors describe the **interaction of on-shell external particles with a gauge-invariant local operator insertion**, which has a non-lightlike momentum q

$$\mathcal{F}_{\mathcal{O}}(p_1, \ldots, p_n; q) = \int d^4x e^{-iqx} \langle p_1, \ldots, p_n | \mathcal{O}(x) | 0 \rangle =$$



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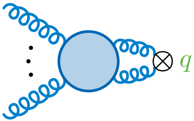
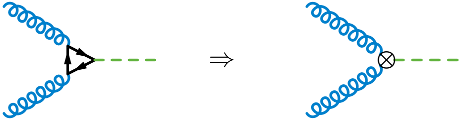


- these objects are just scattering amplitudes with special ‘composite’ particles
- such objects appear when modeling the real world, for instance in the heavy-top limit of QCD



Supersymmetric Form Factors

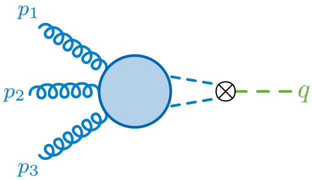
In this talk, we'll stay in the idealized world of planar $\mathcal{N} = 4$ SYM theory, and consider form factors involving the operator $\text{tr}(\phi^2)$

- This quantity first has nontrivial kinematic dependence for $n = 3$:

$$s_{i\ldots j} = (p_i + \cdots + p_j)^2$$

$$u = \frac{s_{12}}{s_{123}}, \quad v = \frac{s_{23}}{s_{123}}, \quad w = \frac{s_{13}}{s_{123}}$$

$$u + v + w = 1$$



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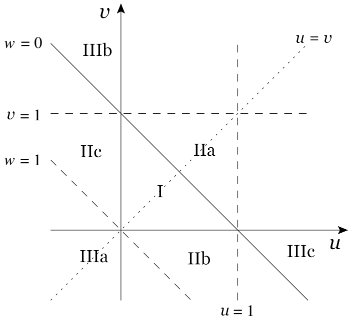

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IIa,b,c = scattering / spacelike operator

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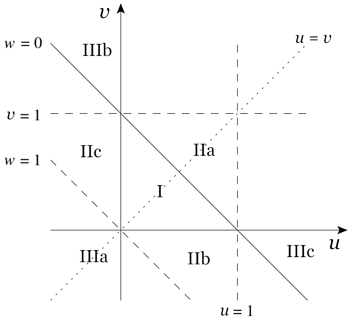

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In the same paper, it was also shown that the answer could be **bootstrapped** using a small number of constraints

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Bootstrap Methods

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- identify the space of functions the form factor is expected to live in
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  - types of functions consistent with expectations

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basis of transcendental functions

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rational coefficients

- require this ansatz to have all the known properties of the form factor, such as symmetries and appropriate behavior in special kinematic limits

$\Rightarrow$ **unique function**

Bootstrapping the Three-Point Form Factor

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- To simplify this problem, we first use the fact that the infrared divergences in these form factors are already completely understood, so we can divide them out:

$$\mathcal{F}_3 = \mathcal{F}_3^{\mathsf{IR}} \exp(R_3)$$

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This reduces the problem of computing the infrared-divergent three-point form factor $\mathcal{F}_3$ to determining the function $R_3^{(L)}$ at each loop order

Analytic Properties of R_3

To formulate an ansatz for $R_3^{(L)}$ at higher loops, we first analyze the two-loop answer

- This function is given by [Brandhuber, Travaglini, Yang (2012)]

$$
\begin{aligned}
R_3^{(2)} = & -2\sum_{i=1}^{3}\left[\mathrm{J}_4\left(-\frac{u_i u_{i+1}}{u_{i+2}}\right) + 4\,\mathrm{Li}_4\left(1-1/u_i\right) + \frac{\ln^4 u_i}{3!}\right] - \frac{\ln^4(uvw)}{4!} \\
& -2\left[\sum_{i=1}^{3}\mathrm{Li}_2(1-1/u_i)\right]^2 + \frac{1}{2}\left[\sum_{i=1}^{3}\ln^2 u_i\right]^2 - \frac{23}{2}\zeta_4
\end{aligned}
$$

where $\{u_1, u_2, u_3\} = \{u, v, w\}$, and

$$
\mathrm{J}_4(t) = \mathrm{Li}_4(t) - \ln(-t)\mathrm{Li}_3(t) + \frac{\ln^2(-t)}{2!}\mathrm{Li}_2(t) - \frac{\ln^3(-t)}{3!}\mathrm{Li}_1(t) - \frac{\ln^4(-t)}{48}
$$

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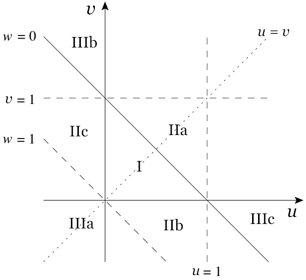
$$\mathcal{S}(R_3^{(2)}) = \sum_{x\in\{u,v,w\}} x \otimes \ldots$$



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> To attempt to bootstrap the three-point form factor, we assume that $R_3^{(L)}$ exists within the space of functions defined by these properties

Bootstrapping Form Factors

We then require a general ansatz of these types of functions to have the expected properties of R_3:

- Dihedral symmetry that exchanges the three on-shell states
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  [Alday, Maldacena (2007)] [Maldacena, Zhiboedov (2010)] [Brandhuber, Spence, Travaglini, Yang (2010)]

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Jointly, these constraints allow us to bootstrap $R_3^{(L)}$ through **eight loops**

Bootstrapping Form Factors

The number of free parameters that remain at each stage in the bootstrap calculation:

L	2	3	4	5	6	7	8
*symbols in $\mathcal{C}$	48	249	1290	6654	34219	????	????
dihedral symmetry	11	51	247	1219	????	????	????
*$(L-1)$ final entries	5	9	20	44	86	191	191
L^{th} discontinuity	2	5	17	38	75	171	164
collinear limit	**0**	1	2	8	19	70	6
OPE $T^2 \ln^{L-1} T$	0	**0**	**0**	4	12	56	**0**
OPE $T^2 \ln^{L-2} T$	0	0	0	**0**	**0**	36	0
OPE $T^2 \ln^{L-3} T$	0	0	0	0	0	**0**	0
OPE $T^2 \ln^{L-4} T$	0	0	0	0	0	0	0
OPE $T^2 \ln^{L-5} T$	0	0	0	0	0	0	0

[Dixon, Gürdoğan, AJM, Wilhelm (2022)]

*incorporate empirical constraints that will be described on the next slide

Surprising Analytic Features

Two surprising types of analytic structure become apparent when one studies $R_3^{(L)}$ to high loop order

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(i) If we 'minimally' normalize the form factor, certain letters never appear next to each other:

$$\mathcal{F}_3 = \mathcal{F}_3^{\text{BDS-like}} F_3 \qquad \Rightarrow \qquad \mathcal{S}(F_3) \not\supset \begin{cases} \ldots \frac{1-u}{u} \otimes \frac{1-v}{v} \ldots \\ \ldots \frac{u}{vw} \otimes \frac{1-u}{u} \ldots \\ \ldots \frac{1-u}{u} \otimes \frac{u}{vw} \ldots \end{cases}$$

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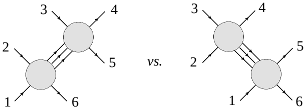
$\Rightarrow$ This resembles the **cluster adjacency relations** that have been observed in amplitudes:
[Steinmann (1960)] [Cahill, Stapp (1975)] [Drummond, Foster, Gürdoğan (2017)] [Caron-Huot, Dixon, Dulat, von Hippel, AJM, Papathanasiou (2019)]



$$\text{Disc}_{s_{234}}(\text{Disc}_{s_{345}}(A_6)) = 0$$

However, it doesn't seem to have the same physical or clustery interpretation

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(ii) Only certain sequences of letters are observed to appear at the end of the symbol

transcendental weight	1	2	3	4	5	6	7	8
naïve number of final entries	6	18	36	72	144	270	510	930
observed number of final entries	3	6	12	24	45	85	155	279

A New Amplitude/Form Factor Duality

It turns out these empirical features can be understood as arising from a **new duality between $\mathcal{F}_3$ and six-particle amplitudes** in planar $\mathcal{N} = 4$ supersymmetric Yang-Mills theory

- Naïvely, these two quantities have nothing to do with each other—the amplitude is a function of three independent variables

$$\hat{u} = \frac{s_{12}s_{45}}{s_{123}s_{345}}, \qquad \hat{v} = \frac{s_{23}s_{56}}{s_{234}s_{123}}, \qquad \hat{w} = \frac{s_{34}s_{61}}{s_{345}s_{234}}$$

  and involves nine symbol letters, some of which depend on the algebraic combination

$$\sqrt{(1 - \hat{u} - \hat{v} - \hat{w})^2 - 4\hat{u}\hat{v}\hat{w}}$$

- On the other hand, this amplitude is has been computed through seven loops using bootstrap techniques, so there's plenty of data to look for robust new relations ...

A New Amplitude/Form Factor Duality

Empirically, we find the surprising relation:

$$F_3^{(L)}(u,v,w) = S\left(A_6^{(L)}(\hat{u},\hat{v},\hat{w})\right)\Big|_{\hat{u}_i \to \hat{u}_i(u,v,w)}$$

[Dixon, Gürdoğan, AJM, Wilhelm (2021)]

where S denotes the **antipode map** that is defined on polylogarithms, and we make the replacements

$$\hat{u}_1 = \hat{u}(u,v,w) = \frac{vw}{(1-v)(1-w)}$$

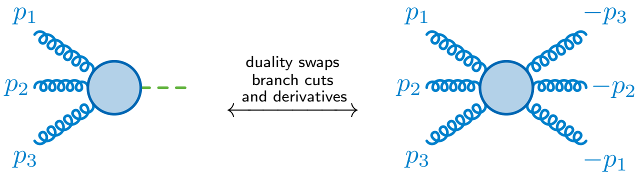
$$\hat{u}_2 = \hat{v}(u,v,w) = \frac{uw}{(1-u)(1-w)}$$

$$\hat{u}_3 = \hat{w}(u,v,w) = \frac{uv}{(1-u)(1-v)}$$

- At symbol level, the antipode map merely reverses the order of integration:

$$S(x_1 \otimes x_2 \otimes \cdots \otimes x_m) = (-1)^m x_m \otimes \cdots \otimes x_2 \otimes x_1$$

A New Amplitude/Form Factor Duality



- The $u+v+w=1$ form factor constraint implies $\sqrt{(1-\hat{u}-\hat{v}-\hat{w})^2-4\hat{u}\hat{v}\hat{w}}=0$, which can be thought of as restricting to a 'twisted forward scattering' configuration
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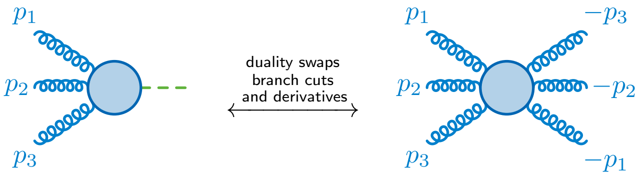

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- Only six symbol letters survive on this kinematic surface
- This duality "explains" the surprising form factor properties:
  - $\Rightarrow$ The extended Steinmann relations obeyed by A_6 imply the adjacency restrictions in F_3
  - $\Rightarrow$ The multiple-final-entry conditions obeyed by F_3 follow from a 'coaction principle' for A_6

    [Caron-Huot, Dixon, Dulat, von Hippel, AJM, Papathanasiou (2019)]

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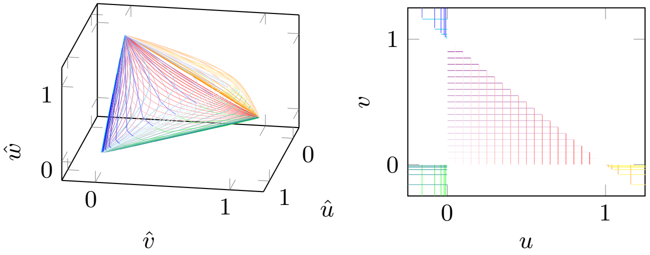

L	number of symbol terms
1	6
2	12
3	636
4	11,208
5	263,880
6	4,916,466
7	92,954,568
8	1,671,656,292

- Explicitly checked through seven loops—exact match on over 92 million terms
- Transcendental constants (such as ζ_3 and ζ_5) also participate in this duality, but not $i\pi$
- Physical interpretation of the antipode map completely obscure... one hint is that collinear and soft limits are exchanged via the duality

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What about higher particle multiplicities?

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- By exploring the properties of this form factor, we find that it obeys a similar but different **antipodal self-duality**:

$$F_4(u_i, v_i)\big|_{\mathsf{tr}_5=0} = S\left(F_4(u_i, v_i)\big|_{\mathsf{tr}_5=0}\right)\big|_{u_i, v_i \to g(u_i), g(v_i)}$$

where the constraint $\mathrm{tr}_5 = 0$ restricts us to parity-even kinematics

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[Dixon, Gürdoğan, Liu, AJM, Wilhelm (2022)]

***now also three loops**

[Dixon, Gürdoğan, Liu, AJM, Wilhelm, to appear]

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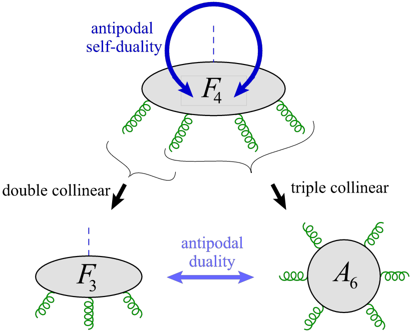

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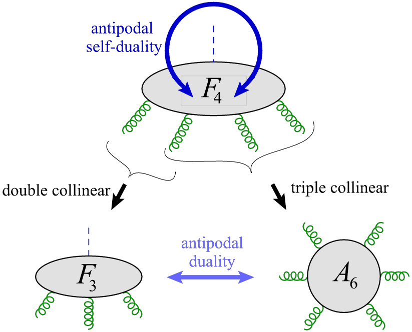

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Does this point to a more extensive web of antipodal relations between amplitudes and form factors at higher particle multiplicity?

Conclusions

Bootstrap techniques can be used to compute quantities to high loop orders in quantum field theory

- These high-loop results give us new insights into **analytic** and **number-theoretic** properties of perturbative QFT

We have also identified a **novel and surprising duality involving form factors and amplitudes**

- What is physics underlying this duality, and can it be extended to all particle multiplicity?
- Can a connection between cluster algebras and form factors be made more directly?
- Has indirect connections to **real-world QCD processes**

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Thanks!

The Antipode

The antipode map S is defined recursively by the condition

$$\mu(S \otimes \text{id})\Delta\big(G(\vec{a}; z)\big) = \mu(\text{id} \otimes S)\Delta\big(G(\vec{a}; z)\big) = 0$$

- At weight one, we just get
$$S(G(a; z)) + G(a; z) = 0$$
- At weight two, we get
$$S(G(a, b; z)) + S(G(a; z))G(b; a) + S(G(b; z))\Big(G(a; z) - G(a; b)\Big) + G(a, b; z) = 0$$
- ...