BRIN MATHEMATICS RESEARCH CENTER

Polylogarithms, Cluster Algebras, and Scattering Amplitudes

September 11-15, 2023

About the Workshop

This workshop seeks to shed light on the mysterious connection between multiple polylogarithms, cluster algebras, and scattering amplitudes by bringing together experts in these seemingly unrelated fields. Other topics to be discussed include Zagier's Polylogarithm conjecture, depth reduction, Hodge correlators, and quantum polylogarithms.

Speakers

Steven Charlton, MPI for Mathematics
Vladimir Fock, Université de Strasbourg
Ömer Gürdoğan, University of Southampton
Richard Hain, Duke University
Minoru Hirose, Nagoya University
Rinat Kashaev, Université de Genève
Ruth Kellerhals, Université de Fribourg
Matilde Lalin, Université de Montréal
Andrew McLeod, University of Copenhagen
Tomoki Nakanishi, Nagoya University
Daniil Rudenko, University of Chicago
Nobuo Sato, National Taiwan University
Marcus Spradlin, Brown University
Cristian Vergu, University of Copenhagen
Lauren Williams, Harvard University
Jianqiang Zhao, The Bishop's School

Organizers

Herbert Gangl, Durham University
Dani Kaufman, Copenhagen University
Zack Greenberg, Heidelberg University
Christian Zickert, University of Maryland

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College Park, MD 20742

DEPARTMENT OF MATHEMATICS

brinmrc.umd.edu

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Schedule at a Glance

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11:00	Coffee Break Marcus Spradlin	Coffee Break Lauren Williams	Coffee Break Daniil Rudenko	Coffee Break Marcus Spradlin	Coffee Break Vladimir Fock
12:00	Lunch	Lunch (on your own)	Lunch	Lunch (on your own)	Lunch
13:00					
14:00	Daniil Rudenko	Marcus Spradlin	Lauren Williams	Daniil Rudenko	
15:00	Cristian Vergu	Andrew McLeod	Jianqiang Zhao	Nobuo Sato	
16:00	Coffee Break Matilde Lalin	Coffee Break Ömer Gürdoğan	Coffee Break Steven Charlton	Coffee Break Minoru Hirose	
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Workshop Overview

The classical polylogarithms were studied in the 18th and 19th century by many prominent mathematicians including Abel, Euler, Kummer and Lobachevsky. They were mainly interested in special values and functional relations. In the 20th century deep relations to algebraic K-theory, characteristic classes and motivic cohomology were discovered for the dilogarithm, and conjectural generalizations were formulated. In the 21st century it was discovered that formulas for scattering amplitudes often involve polylogarithms evaluated at cluster coordinates for a Grassmannian. This brings the theory of cluster algebras to the study of polylogarithms. There are many exciting recent developments including the proof of Zagier's conjecture (expressing the regulator in terms of classical polylogarithms) in weight 4 by Goncharov and Rudenko (2018) following a depth reduction formula by Gangl (2016), the general depth reduction (to half the weight) by Rudenko (2020, formerly a conjecture of Goncharov), the precise formulation of cluster polylogarithms and depth reduction in weight 6 by Matveiakin and Rudenko (2022), a cluster formulation of the second motivic Chern class by Goncharov and Kislinskyi (2021), and iterated integral expressions for Grassmannian and Aomoto polylogarithms by Charlton, Gangl and Radchenko (2019).

This workshop seeks to shed light on the mysterious connection between multiple polylogarithms, cluster algebras, and scattering amplitudes by bringing together experts in these seemingly unrelated fields. Other topics to be discussed include Zagier's Polylogarithm Conjecture, depth reduction, Hodge correlators, and quantum polylogarithms.

Organizing committee

Dani Kaufman, Copenhagen University

Herbert Gangl, Durham University / MPI Bonn

Zack Greenberg, Heidelberg University

Christian Zickert, University of Maryland

Workshop Schedule

Monday, September 11, 2023

9:00 - 9:25	Breakfast
9:25 - 9:30	Sandra Cerrai (University of Maryland) *Opening*
9:30 - 10:20	Richard Hain (Duke University) *Polylogs, Prehistory and Future Directions*
10:20 - 10:50	Coffee Break
10:50 - 11:40	Marcus Spradlin (Harvard/Brown University) *Cluster Algebras and Polylogarithms-1*
12:00 - 1:30	Lunch
1:30 - 2:20	Daniil Rudenko (University of Chicago) *Lecture 1: The Hopf Algebra of Multiple Polylogarithms*
2:20 - 2:30	Break
2:30 - 3:20	Cristian Vergu (University of Copenhagen (Niels Bohr Institute)) *Configuration Spaces and Singularities of Polylogarithms Arising from Feynman Integrals*
3:20 - 3:50	Coffee Break
3:50 - 4:40	Matilde Lalin (Université de Montréal) *Evaluations of Areal Mahler Measure and Polylogarithms*
4:40 - 6:00	High Tea

Tuesday, September 12, 2023

9:00 - 9:30 Breakfast

9:30 - 10:20 Ruth Kellerhals (Université de Fribourg)
Trilogarithms and Volumes of Hyperbolic 5-Manifolds

10:20 - 10:50 Coffee Break

10:50 - 11:40 Lauren Williams (Harvard University)
Cluster Algebras and the Amplituhedron-1

12:00 - 1:30 Lunch (on your own)

1:30 - 2:20 Marcus Spradlin (Harvard/Brown University)
Cluster Algebras and Polylogarithms-2

2:20 - 2:30 Break

2:30 - 3:20 Andrew McLeod (University of Copenhagen (Niels Bohr Institute))
An Antipodal Amplitude/Form Factor Duality

3:20 - 3:50 Coffee Break

3:50 - 4:40 Ömer Gürdoğan (University of Southampton)
Cluster-adjacent A_n polylogarithms

Wednesday, September 13, 2023

9:00 - 9:30 Breakfast

9:30 - 10:20 Rinat Kashaev (Université de Genève)
Quantum Dilogarithms Associated with Local Fields

10:20 - 10:50 Coffee Break

10:50 - 11:40 Daniil Rudenko (University of Chicago)
Lecture 2: The Depth Conjecture

12:00 - 1:30 Lunch

1:30 - 2:20 Lauren Williams (Harvard University)
Cluster Algebras and the Amplituhedron-2

2:20 - 2:30 Break

2:30 - 3:20 Jianqiang Zhao (The Bishop's School, San Diego County)
Alternating Multiple Mixed Values

3:20 - 3:50 Coffee Break

3:50 - 4:40 Steven Charlton (MPI for Mathematics, Bonn)
New Polylogarithm Depth Reductions in Weight 5 and 6 (with a View Towards Zagier's Polylogarithm Conjecture)

7:00 - 9:00 Dinner

Thursday, September 14, 2023

8:30 - 9:10	Breakfast
9:10 - 10:00	Lauren Williams (Harvard University) *Cluster Algebras and the Amplituhedron-3*
10:00 - 10:30	Coffee Break
10:30 - 11:20	Marcus Spradlin (Harvard/Brown University) *Cluster Algebras and Polylogarithms-3*
11:40 - 1:30	Lunch (on your own)
1:30 - 2:20	Daniil Rudenko (University of Chicago) *Lecture 3: Cluster Polylogarithms*
2:20 - 2:30	Break
2:30 - 3:20	Nobuo Sato (National Taiwan University) *On Iterated Beta Integrals*
3:20 - 3:50	Coffee Break
3:50 - 4:40	Minoru Hirose (Nagoya University) *Iterated Integrals along Loops and Cyclic Sum Formula*

Friday, September 15, 2023

9:00 - 9:30 Breakfast

9:30 - 10:20 Tomoki Nakanishi (Nagoya University)
Dilogarithm Identities, Cluster Algebras, and Cluster Scattering Diagrams

10:20 - 10:50 Coffee Break

10:50 - 11:40 Vladimir Fock (Université de Strasbourg)
Quantum Dilogarithm as a Wave Function for Integrable Systems

12:00 - 1:30 Lunch

Abstracts of talks

Polylogs, Prehistory and Future Directions

Richard Hain

Duke University

Monday, September 11, 2023 @ 9:30 AM

This talk will begin with an overview of the early days (1970s and 1980s) of the modern study of polylogarithms. I'll begin by reviewing the relationship between group homology and algebraic K-theory and the construction of various incarnations of the first two Chern classes from cohomology classes on general linear groups constructed from the logarithm and the dilogarithm (work of Bloch, Beilinson and Deligne). I will also discuss efforts to extend this to higher Chern classes and higher polylogs such as Grassmann polylogs and Goncharov's breakthrough work on the trilogarithm and the third Chern class around 1990. The recent remarkable work of Goncharov and Rudenko (which I will not discuss) extends the story to the 4th Chern class and the 4-logarithm.
The increasing complexity of the story suggests that a new conceptual framework is needed. For Goncharov, this appears to be his Hodge correlators. A central part of this story is a certain cyclic construction and a related Lie coalgebra, special cases of which appear in his dihedral symmetry paper. In the second part of this talk, I will explain how that structure arises from the Goldman–Turaev Lie bialgebra of a hyperbolic surface. This is the rational vector space spanned by the conjugacy classes in the fundamental group of the surface. It carries a natural bracket (due to Goldman) and a cobracket (due to Turaev). The cobracket depends on the choice of a framing of the surface. The GT-Lie bialgebra carries a natural mixed Hodge structure (so it is naturally isomorphic to its associated weight graded) and Goncharov's combinatorial constructions correspond to the bracket and cobracket on the associated graded.

Cluster Algebras and Polylogarithms-1

Marcus Spradlin

Harvard/Brown University

Monday, September 11, 2023 @ 10:50 AM

I will give an introduction to polylogarithms and Grassmannian cluster algebras from the viewpoint of their appearance in scattering amplitudes computed in quantum field theory.

Lecture 1: The Hopf Algebra of Multiple Polylogarithms

Daniil Rudenko

University of Chicago

Monday, September 11, 2023 @ 1:30 PM

I am going to give three lectures about polylogarithms and their relation with cluster algebras.
1. Hopf algebra of multiple polylogarithms.
2. Depth Conjecture.
3. Cluster polylogarithms.

Configuration Spaces and Singularities of Polylogarithms Arising from Feynman Integrals

Cristian Vergu

University of Copenhagen (Niels Bohr Institute)

Monday, September 11, 2023 @ 2:30 PM

Feynman integrals in quantum field theory are a natural source of polylogarithmic functions. The singularities of Feynman integrals correspond to the branch points of the polylogarithms they evaluate to. One can evaluate the singularities of a given Feynman integral, without computing it, by solving a set of equations called Landau equations.
The Landau equations also have a less familiar geometric interpretation. To describe it, one has to construct so-called *on-shell spaces*, which are spaces obtained by imposing some constraints on the integration variables of a given Feynman integral. These on-shell spaces can be conveniently described as configurations of points (possibly with extra constraints). Then, the Landau equations are the conditions that a projection map, restricted to these on-shell spaces, has critical points.
I will discuss a number of examples arising from physics where the geometric interpretation has been useful.

Evaluations of Areal Mahler Measure and Polylogarithms

Matilde Lalin

Université de Montréal

Monday, September 11, 2023 @ 3:50 PM

The (logarithmic) Mahler measure of a non-zero rational function P in n variables is defined as the mean of $\log|P|$ (with respect to the normalized arclength measure) restricted to the standard n-dimensional unit torus. It has been related to special values of L-functions via polylogarithms. Pritsker (2008) defined a natural counterpart of the Mahler measure, which is obtained by replacing the normalized arclength measure on the standard n-torus by the normalized area measure on the product of n open unit disks. In this talk, we will investigate some similarities and differences between the two versions of Mahler measure. We will also discuss some evaluations of the areal Mahler measure of multivariable polynomials, which also yields special values of L-functions via polylogarithms. This is joint work with Subham Roy.

Trilogarithms and Volumes of Hyperbolic 5-Manifolds

Ruth Kellerhals

Université de Fribourg

Tuesday, September 12, 2023 @ 9:30 AM

Polylogarithms arise as characteristic volume functions in non-Euclidean geometry in a natural way, at least in smaller dimensions. We explain this aspect and concentrate then on the hyperbolic case. We present some results about volumes of hyperbolic space forms of dimension five and the appearance of certain modified Trilogarithms. At the end, we mention some open problems, also in view of higher dimensions.

Cluster Algebras and the Amplituhedron-1

Lauren Williams

Harvard University

Tuesday, September 12, 2023 @ 10:50 AM

I will give an introduction to cluster algebras, a remarkable class of commutative rings developed by Fomin and Zelevinsky to study total positivity. I will then explain how cluster algebras are connected to the amplituhedron, a geometric object defined by Arkani-Hamed and Trnka to study scattering amplitudes in N=4 super Yang Mills theory.

Cluster Algebras and Polylogarithms-2

Marcus Spradlin

Harvard/Brown University

Tuesday, September 12, 2023 @ 1:30 PM

I will give an introduction to polylogarithms and Grassmannian cluster algebras from the viewpoint of their appearance in scattering amplitudes computed in quantum field theory.

An Antipodal Amplitude/Form Factor Duality

Andrew McLeod

University of Copenhagen (Niels Bohr Institute)

Tuesday, September 12, 2023 @ 2:30 PM

Traditional methods for computing quantities such as scattering amplitudes and form factors in quantum field theory become intractable at high perturbative orders. However, a great deal is now known about the mathematical properties of these quantities, especially in supersymmetric gauge theories. In this talk, I will describe how this knowledge can be leveraged to 'bootstrap' amplitudes and form factors directly, by constructing an ansatz with the appropriate mathematical structure and requiring it to have certain expected behavior in special kinematic limits. I will focus on the example of three-point form factors in maximally supersymmetric gauge theory, which have recently been bootstrapped through eight loops. I will then describe a remarkable new duality between this form factor and six-particle amplitudes in the same theory, which holds order by order in perturbation theory.

Cluster-adjacent A_n polylogarithms

Ömer Gürdoğan

University of Southampton

Tuesday, September 12, 2023 @ 3:50 PM

Cluster adjacency is a restriction that can be imposed on spaces of polylogarithms whose symbol letters are $\mathcal{A}$ coordinates of a cluster algebra. It is motivated by observations on scattering ampli-tudes, where the symbol letters are G $r(4, n)$ $\mathcal{A}$ coordinates, and requires that all pairs of adjacent symbol letters are cluster compatible.

Even if other properties that define a scattering amplitude do not straightforwardly translate from G $r(4, n)$ to G $r(2, n)$, I will describe various analogues with cluster-adjacency properties that can be defined. I will state conjectures on generating functions that count their dimensions, and linear relations that are satisfied by these generating functions.

Quantum Dilogarithms Associated with Local Fields

Rinat Kashaev

Université de Genève

Wednesday, September 13, 2023 @ 9:30 AM

Based on experience with the Teichmuller TQFT, one can give a definition of a quantum dilogarithm over a locally compact Abelian group endowed with a non-degenerate complex unit circle valued quadratic form. I will talk about an example of such a notion associated to any local field F. By using the similar techniques as for the Teichmuller TQFT, one can construct generalised distribution valued 3d TQFTs. The associated 3-manifold invariants are expected to be enumerative invariants counting with specific weights representations of the fundamental group π_1 into the group PSL_2F. This is the work in collaboration with Stavros Garoufalidis.

Lecture 2: The Depth Conjecture

Daniil Rudenko

University of Chicago

Wednesday, September 13, 2023 @ 10:50 AM

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Cluster Algebras and the Amplituhedron-2

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Alternating Multiple Mixed Values

Jianqiang Zhao

The Bishop's School, San Diego County

Wednesday, September 13, 2023 @ 2:30 PM

In this talk, we define and study a variant of multiple zeta values (MZVs) of level four, called alternating multiple mixed values (AMMVs), forming a $\mathbf{Q}(i)$-subspace of the special values of multiple polylogarithms at fourth roots of unity. This variant includes the alternating version of Hoffman's multiple t-values, Kaneko-Tsumura's multiple T-values, and the multiple S-values studied by the authors previously as special cases. We exhibit nice properties of AMMVs similar to the ordinary MZVs such as the duality, integral shuffle and series stuffle relations and then establish some other explicit relations among them. We will also discuss some conjectures concerning the dimensions of the above-mentioned subspaces of AMMVs. These conjectures hint at a few very rich but previously overlooked algebraic and geometric structures associated with these vector spaces. This is a joint work with Ce Xu and Lu Yan.

New Polylogarithm Depth Reductions in Weight 5 and 6 (with a View Towards Zagier's Polylogarithm Conjecture)

Steven Charlton

MPI for Mathematics, Bonn

Wednesday, September 13, 2023 @ 3:50 PM

Goncharov sketched a programme to tackle Zagier's Polylogarithm Conjecture on $\zeta_F(m)$ by understanding the structure of multiple polylogarithms in weight m, in particular how the motivic framework should provide a characterisation of the depth of a multiple polylogarithm by a filtration arising from iterating the coproduct/cobracket. In weights 2 and 3, this is essentially equivalent to the result that one can write every multiple polylogarithm in terms of Li_2 and Li_3 respectively. In weight 4 however, the function $\mathrm{Li}_{3,1}$ (or $I_{3,1}$ as an integral) is genuinely of depth 2 and cannot be reduced, but the framework predicts that $I_{3,1}$(dilogarithm 5-term relation, z) should reduce. In 2011, Gangl gave this reduction explicitly, and provided 122 Li_4 terms (whose arguments typically involved structured products of up to 4 cross-ratios) found with perspicacious experimentation and computer assistance; a conceptual derivation was given later, in 2018, by Goncharov and Rudenko as a consequence of a beautiful and simple weight 4 identity, with a cluster-geometric flavour. Since then various subsets of Matveiakin, Rudenko, Gangl, Radchenko, and myself, have worked to extend these cluster-geometric identities, and in particular the consequent depth reduction identities, to higher weight. I will report on the progress, so far, of the known depth reductions in weight 5 and weight 6, what is still left for us to do, and what this means for trying to tackle $\zeta_F(5)$ and $\zeta_F(6)$.

Cluster Algebras and the Amplituhedron-3

Lauren Williams

Harvard University

Thursday, September 14, 2023 @ 9:10 AM

I will give an introduction to cluster algebras, a remarkable class of commutative rings developed by Fomin and Zelevinsky to study total positivity. I will then explain how cluster algebras are connected to the amplituhedron, a geometric object defined by Arkani-Hamed and Trnka to study scattering amplitudes in N=4 super Yang Mills theory.

Cluster Algebras and Polylogarithms-3

Marcus Spradlin

Harvard/Brown University

Thursday, September 14, 2023 @ 10:30 AM

I will give an introduction to polylogarithms and Grassmannian cluster algebras from the viewpoint of their appearance in scattering amplitudes computed in quantum field theory.

Lecture 3: Cluster Polylogarithms

Daniil Rudenko

University of Chicago

Thursday, September 14, 2023 @ 1:30 PM

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On Iterated Beta Integrals

Nobuo Sato

National Taiwan University

Thursday, September 14, 2023 @ 2:30 PM

In this talk, I will explain my recent work with Minoru Hirose on iterated beta integrals. Iterated beta integrals are certain types of iterated integrals over the universal abelian covering spaces of punctured complex projective lines. The iterated beta integrals contain hyperlogarithms and beta integral as special cases, and satisfies a generalization of Goncharov's differential equation under a certain setup. They also enjoy a new feature, which we call "translation invariance" property. This translation invariance property gives equalities between iterated integrals over finite coverings of punctured projective lines. Especially, if the coverings are of genus zero, the equality gives non-trivial relations between hyperlogarithms. Zagier's 2-3-2 formula for multiple zeta values and Zhao's 2-1 formulas for multiple zeta star values, together with their variances are special instances of the equalities that comes from the translation invariance. What is even more exciting is that it also gives brand new interesting formulas, one of which explains a recent conjecture by Charlton.

Iterated Integrals along Loops and Cyclic Sum Formula

Minoru Hirose

Nagoya University

Thursday, September 14, 2023 @ 3:50 PM

In my talk, I will start with introducing iterated integrals along loops on punctured Riemann spheres. Here, a loop means a homotopy class of continuous maps from the unit circle to the punctured Riemann sphere. Then, we will talk about a certain generalization of the cyclic sum formula for multiple zeta values by Hoffman and Ohno to a larger class of identities between hyperlogarithms, which we call the sandwiched cyclic sum formula. Finally, we will discuss the relationship between the iterated integrals along loops and the sandwiched cyclic sum formula. All results in this talk are joint works with Nobuo Sato at National Taiwan University.

Dilogarithm Identities, Cluster Algebras, and Cluster Scattering Diagrams

Tomoki Nakanishi

Nagoya University

Friday, September 15, 2023 @ 9:30 AM

I review the intriguing relation between dilogarithm identities and cluster algebras, which is recently updated in view of cluster scattering diagrams. In particular, I explain that any dilogarithm identity associated with a period in a cluster algebra or a cluster scattering diagram is reduced to a trivial one by applying the pentagon relation (the five-term identity) possibly infinitely many times.

Quantum Dilogarithm as a Wave Function for Integrable Systems

Vladimir Fock

Université de Strasbourg

Friday, September 15, 2023 @ 10:50 AM

A tame symbol is a bimultiplicative 2-cocycle on the group of nonvanishing functions a circle given by an explicit formula. The tame symbol is related to Heisenberg group, resultant, Witt ring, Gauss reciprocity and many other subjects. We will use the tame symbol to define a homology class (with values in the multiplicative group) of a Lagrangian subvariety of a cluster A-variety. Say that a Lagrangian subvariety is Bohr-Sommerfeld if this class is trivial. We will show in the example of dimension 2 that every Bohr-Sommerfeld curve gives a solution of a difference equation, which is a quantization of the equation of the curve with quantization parameter equal to 1 (and conjecturally for rational quantisation parameters). The solution is given by quantum dilogarithms of algebraic functions.

The Brin Mathematics Research Center

The Brin Mathematics Research Center is a research center that sponsors activity in all areas of pure and applied mathematics and statistics. The Brin MRC was funded in 2022 through a generous gift from the Brin Family. The Brin MRC is part of the Department of Mathematics at the University of Maryland, College Park.

Activities sponsored by the Brin MRC include long programs, conferences and workshops, special lecture series, and summer schools. The Brin MRC provides ample opportunities for short-term and long-term visitors that are interested in interacting with the faculty at the University of Maryland and in experiencing the metropolitan Washington DC area.

The mission of the Brin MRC is to promote excellence in mathematical sciences. The Brin MRC is home to educational and research activities in all areas of mathematics. The Brin MRC provides opportunities to the global mathematical community to interact with researchers at the University of Maryland. The center allows the University of Maryland to expand and showcase its mathematics and statistics research excellence nationally and internationally.

List of Participants

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Steven Charlton, MPI for Mathematics, Bonn
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Julian Kaufmann, Notre Dame University
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