Hints for descent questions on Problem Sheet 1.

4(ii) Consider the smallest solution (X, Y, Z) of the given equation with XYZ non-zero (you can easily rule out the case where one becomes 0). Then look modulo 5, keeping in mind the possible fourth powers of integers mod 5. Now you can deduce a certain divisibility result for Z. [This should allow you to reduce the solution to a smaller one.]

6(i) First consider the case $N = 2$. Assume for a contradiction that $\sqrt{2}$ is rational, hence equal to some quotient p/q, say, with smallest positive integers p and q; then square both sides and consider divisibility by powers of 2. Can you reduce that to a similar equation to write 2 as a quotient of two squares?

6(ii) The second descent question is really hard (whence the (*)).
Hints:

- use the knowledge on Pythagorean triples (re-write them with new parameters $(2uv, u^2 - v^2, u^2 + v^2)$, and with x and y, respectively);
- deduce the parity of b and c (they are implicitly determined!);
- both b and c have two presentations.

You may have gotten to this point–but then a crucial observation is:

- decompose uv (hence xy) into 4 smaller building blocks and try to find two new triples in terms of these (you may have to distinguish two cases).