Review 1: The Box Integral

Let x_1, x_2, x_3, x_4 be four points in $\mathbb{R}^4$. Then

$$\int \frac{d^4x}{(x-x_1)^2(x-x_2)^2(x-x_3)^2(x-x_4)^2} = \frac{2\pi^2}{\sqrt{|\det Q|}}\mathrm{vol}(Q)$$

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$$z\bar{z} = u \qquad (1-z)(1-\bar{z}) = v$$

$$2\,\mathrm{vol}(Q) = \mathrm{Li}_2\left(\frac{z}{\bar{z}}\right) - \mathrm{Li}_2\left(\frac{1-z}{1-\bar{z}}\right) + \mathrm{Li}_2\left(\frac{1-1/z}{1-1/\bar{z}}\right) - (z \leftrightarrow \bar{z})$$

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Caution: in contrast to what follows shortly, in this example the x_i are generic; I am NOT imposing that $(x_i - x_{i+1})^2 = 0$.

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$\mathrm{vol}(Q)$ is the volume of the ideal hyperbolic tetrahedron with

dihedral angles

$$\cos(\angle H_i H_j) = \frac{Q^{-1}_{ij}}{\sqrt{Q^{-1}_{ii} Q^{-1}_{jj}}}$$

Review 2: The Configuration Space for SYM Theory

n-particle scattering amplitudes in "maximally supersymmetric Yang-Mills theory" are naturally functions of n ordered points in $\mathbb{P}^3$, modulo the action of PGL_4. This is the space called Y^4_{n-5} by Hain and MacPherson.

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Since no cross-ratios exist for $n < 6$ the simplest non-trivial amplitude involves $n = 6$ particles, and is a function of the three independent cross-ratios

$$u_1 = \frac{(x_1 - x_3)^2 (x_4 - x_6)^2}{(x_3 - x_6)^2 (x_4 - x_1)^2} \quad u_2, u_3 = \text{cycle} \tag{1}$$

(Here $(x_i - x_{i+1})^2 = 0$!)

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An analytic result for a two-loop 6-particle amplitude was posted on arxiv on **27 November 2009** by Del Duca, Duhr and Smirnov. Their result is expressed as a long linear combination of Goncharov polylogarithms of uniform transcendentality weight 4.