

Proof of (b) Put $\Phi(t) = \frac{a(1-t)}{t(1-a)}$, then $\deg(\Phi) = 1$, $1 - \Phi(t) = \frac{t-a}{t(1-a)}$ and $\frac{\Phi(t)-\Phi(b)}{\Phi(t)} = 1 - \frac{t(1-b)}{b(1-t)} = \frac{b-t}{b(1-t)}$. The following lines contain the proof of the five term relation using lemma 2.2. It is given in two different notations, on the upper lines there is given a graph corresponding to the cycle underneath. Each of the cycles involved is easily seen to be in the graph notation. The equality sign is understood up to negligible terms. We start with the cycle $C_{\Phi(b)}$ which is then decomposed after.

$$\begin{aligned}
& \begin{array}{c} \boxed{1} \text{---} \textcircled{2} \boxed{3} \text{---} \textcircled{1} \text{---} \boxed{2} \text{---} \textcircled{3} \text{---} \bullet \\ 1 \quad \infty \quad \frac{a}{a-1} \quad 0 \quad \Phi(b) \end{array} = \begin{array}{c} \boxed{1} \text{---} \textcircled{2} \boxed{3} \text{---} \textcircled{1} \text{---} \boxed{2} \text{---} \textcircled{3} \text{---} \bullet \\ a \quad 0 \quad \infty \quad 1 \quad b \end{array} = \begin{array}{c} \boxed{1} \text{---} \textcircled{2} \boxed{3} \quad \bullet \text{---} \textcircled{1} \text{---} \boxed{2} \text{---} \textcircled{3} \text{---} \bullet \\ a \quad 0 \quad \infty \quad 1 \quad b \end{array} + \\
& C_{\Phi(b)} = \left[t, 1-t, \frac{t-\Phi(b)}{t} \right] = \left[\frac{a(1-t)}{t(1-a)}, \frac{t-a}{t(1-a)}, \frac{b-t}{b(1-t)} \right] = \left[\frac{1-t}{1-a}, \frac{t-a}{t(1-a)}, \frac{b-t}{b(1-t)} \right] + \\
& = \begin{array}{c} \boxed{1} \text{---} \textcircled{2} \text{---} \bullet \quad \bullet \text{---} \textcircled{1} \text{---} \boxed{3} \text{---} \textcircled{3} \text{---} \bullet \\ a \quad 0 \quad \infty \quad 1 \quad b \end{array} + \begin{array}{c} \boxed{1} \text{---} \textcircled{2} \text{---} \bullet \quad \bullet \text{---} \textcircled{1} \text{---} \boxed{2} \\ a \quad 0 \quad \infty \quad 1 \quad b \end{array} + \begin{array}{c} \boxed{1} \text{---} \textcircled{2} \boxed{3} \text{---} \textcircled{1} \text{---} \boxed{2} \quad \bullet \text{---} \textcircled{3} \text{---} \bullet \\ a \quad 0 \quad \infty \quad 1 \quad b \end{array} + \\
& = \left[\frac{1-t}{1-a}, \frac{t-a}{t(1-a)}, \frac{b-t}{b(1-t)} \right] + \left[\frac{1-t}{1-a}, \frac{t-a}{t(1-a)}, \frac{1}{b} \right] + \left[\frac{a}{t}, \frac{t-a}{t}, \frac{b-t}{b(1-t)} \right] + \\
& = \begin{array}{c} \boxed{1} \text{---} \textcircled{2} \text{---} \boxed{3} \text{---} \textcircled{1} \text{---} \boxed{2} \text{---} \textcircled{3} \text{---} \bullet \\ a \quad 0 \quad \infty \quad 1 \quad b \end{array} + \begin{array}{c} \boxed{1} \quad \bullet \text{---} \textcircled{2} \text{---} \boxed{3} \text{---} \textcircled{1} \text{---} \boxed{2} \text{---} \textcircled{3} \text{---} \bullet \\ a \quad 0 \quad \infty \quad 1 \quad b \end{array} + \begin{array}{c} \boxed{1} \text{---} \textcircled{2} \text{---} \boxed{3} \text{---} \textcircled{1} \text{---} \boxed{2} \text{---} \textcircled{3} \text{---} \bullet \\ a \quad 0 \quad \infty \quad 1 \quad b \end{array} + \\
& = \left[\frac{1-t}{1-a}, \frac{t-a}{1-a}, \frac{b-t}{1-t} \right] + \left[\frac{1-t}{1-a}, \frac{1}{t}, \frac{b-t}{1-t} \right] + \left[\frac{a}{t}, \frac{t-a}{t}, \frac{b-t}{b} \right] +
\end{aligned}$$

Now note that the cycles in the last line can again be reparametrized to give another combination of distinguished

$$C_{\frac{1-b}{1-a}} - C_{1-b} + C_{\frac{a}{b}} - C_a = \rho_2 \left(\left[\frac{1-b}{1-a} \right] - [1-b] - \left[\frac{a}{b} \right] - [a] \right)$$

(after decomposing the second cycle in the first coordinate and discarding a negligible term $[1-a, \dots, \dots]$). This completes the proof.

Table 1: Proof of the five term relation