§2 The action principle

We will start with the Lagrangian formulation. The underlying physical principle behind this formulation can be traced back to the idea that for some physical processes, the natural answer to the question "what is the trajectory that a particle follows" is something like "the most efficient one". Our goal in this section is to understand, in a precise sense, how to characterize this notion of efficiency.

A fundamental example is the free particle in flat space: its motion is along a straight path. What makes the straight path special? The answer is well known: the straight path is the one that minimizes the distance travelled between the origin and the destination of a path. This is equivalent to saying that the motion of the particle is along a trajectory that, assuming constant speed for the particle, minimizes the total time travelled.

This second formulation connects with *Fermat's principle*, which states that the path that a ray of light takes, when moving on a medium, is the one that minimizes the time spent by the light beam. Or more precisely, one should impose that the time is stationary (we will define this precisely below) under small variations of the path.

These two examples suggest a natural question: is there always some quantity, in problems of classical mechanics, that is minimized along physical motion? The answer is that there is indeed such a quantity, known as the *action*. We will now explain how to determine the equations of motion from the action, and then determine the form of the action that reproduces classical Newtonian physics.

Our basic tool will be the "Calculus of variations", which we now describe.