2. A system is described by the Lagrangian

$$L = \frac{1}{2}\dot{q}_1^2 + \frac{1}{2}q_1^2\left(\dot{q}_2^2 + \dot{q}_3^2\right) - \frac{1}{2}(q_2^2 + q_3^2).$$

Find the equations of motion for the system, and use these to show that $q_1^2\left(\dot{q}_2 q_3 - \dot{q}_3 q_2\right)$ is conserved.

<u>Euler-Lagrange:</u>

$$\frac{d}{dt}(\dot{q}_1) - q_1\left(\dot{q}_2^2 + \dot{q}_3^2\right) = 0$$

$$\frac{d}{dt}\left(q_1^2 \dot{q}_2\right) - (-q_2) = 0$$

$$\frac{d}{dt}\left(q_1^2 \dot{q}_3\right) + q_3 = 0$$

$$\frac{d}{dt}\left(q_1^2\left(\dot{q}_2 q_3 - \dot{q}_3 q_2\right)\right) = \frac{d}{dt}\left(q_1^2\dot{q}_2\right)q_3 + q_1^2\dot{q}_2\dot{q}_3 - \frac{d}{dt}\left(q_1^2\dot{q}_3\right)q_2 - q_1^2\dot{q}_3\dot{q}_2$$

$$= -q_2 q_3 + q_1^2\dot{q}_2\dot{q}_3 - (-q_3)q_2 - q_1^2\dot{q}_3\dot{q}_2$$

$$= 0$$