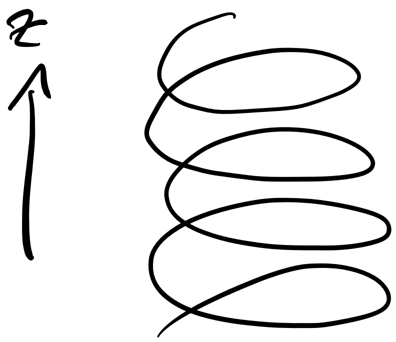
1. A pointlike bead of mass m moves under gravity on a smooth wire bent into the shape of a helix described parametrically by $(x, y, z) = (\cos\theta, \sin\theta, \theta)$ with the z-axis pointing vertically upwards.

   (a) Find the kinetic energy of the particle in terms of θ?



$$T = \frac{1}{2} m \left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right)$$

$$\dot{x} = -\sin(\theta)\,\dot{\theta}$$

$$\dot{y} = \cos(\theta)\,\dot{\theta}$$

$$\dot{z} = \dot{\theta}$$

$$\Rightarrow T = \frac{1}{2} m \left(\dot{\theta}^2 + \dot{\theta}^2\right) = m\dot{\theta}^2$$

(b) Find the Lagrangian of the particle taking θ as the generalised co-ordinate.

$$L = T - V = m\dot{\theta}^2 - (mg\overset{\theta}{z})$$

$$= m\dot{\theta}^2 - mg\theta$$

(c) Find the Euler-Lagrange equation for the particle.

$$0 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = \frac{d}{dt}(2m\dot{\theta}) - (-mg)$$

$$= 2m\ddot{\theta} + mg$$

$$\Updownarrow$$

$$\ddot{\theta} = -g/2$$

(d) Obtain the general solution to the Euler-Lagrange equation.

$$\ddot{\theta} = -g/2 \Rightarrow \theta = \frac{1}{2}(-g/2)\,t^2 + At + B$$

(e) How long does the particle takes to travel from rest at $\theta = 2\pi$ to $\theta = 0$.

Choose the starting time to be $t=0$.

"Rest" = the velocity vanishes.

$$\dot\theta(t=0) = 0$$
$$\theta(t=0) = 2\pi$$

$$\Rightarrow \dot\theta(t=0) = \frac{1}{2}\left(-\frac{g}{2}\right)\cdot 2\cdot \overset{0}{\overset{\shortparallel}{t}} + A = A = 0$$

$$\Rightarrow A = 0$$

$$\Rightarrow \frac{1}{2}\left(-\frac{g}{2}\right)\underbrace{t^2}_{=0} + \underbrace{A}_{=0} t + B = 2\pi \text{ at } t=0$$

$$\Downarrow$$

$$B = 2\pi$$

$$\theta(t) = \frac{1}{2}\left(-\frac{g}{2}\right)t^2 + 2\pi = -\frac{g}{4}t^2 + 2\pi$$

$\theta = 0$ when $-\frac{g}{4}t^2 + 2\pi = 0$

$$\Updownarrow$$

$$t^2 = \frac{8\pi}{g} \Leftrightarrow t = \sqrt{\frac{8\pi}{g}}$$