Lagrangian Mechanics - Generalised coordinates

November 8, 2021

---

1. A system is described by the Lagrangian

$$L = \frac{1}{2}\dot{q}_1^2 + \frac{1}{2}\dot{q}_2^2 + \frac{1}{2}(q_1^2 + q_2^2).$$

Find the equations of motion for the system, and use these to show that

$$Q = \dot{q}_1 q_2 - \dot{q}_2 q_1$$

is conserved. (That is, that $\frac{dQ}{dt} = 0$.)

2. Spherical coordinates on $\mathbb{R}^3$ are given by

$$\begin{aligned} x &= r\sin(\theta)\cos(\varphi) \\ y &= r\sin(\theta)\sin(\varphi) \\ z &= r\cos(\theta) \end{aligned}$$

Show that the kinetic energy of a particle of mass m moving on $\mathbb{R}^3$ is given in these coordinates by

$$T = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin(\theta)^2\dot{\varphi}^2)\,.$$

3. A 'spherical pendulum' consists of a mass m attached to one end of a massless rigid rod of length L which is free to pivot in three dimensions about its other end, which we will take to be attached to the origin of $\mathbb{R}^3$. Assume that the mass is affected by gravity, with a gravitational potential of the form $V = mgz$. Write down the Lagrangian for the pendulum in terms of the spherical coordinates (θ, φ) introduced in the previous problem, and write down the corresponding equations of motion. (You do not need to solve the resulting differential equations.)