Quantum Computing Epiphany Assignment 3

Q1 Consider a two-qubit system. We wish to construct a circuit to realise the operation

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

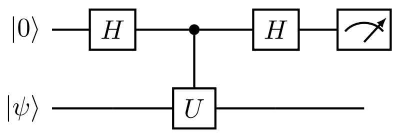
1.1 First decompose this operator as a product of unitary operators which each act non-trivially only on a two-dimensional subspace of the Hilbert space.

1.2 Use the Gray code construction to convert the operators which do not act on a subspace corresponding to a single qubit into ones that do.

1.3 Draw the resulting quantum circuit, making any obvious simplifications.

Q2 Suppose we have a unitary operator U on a one-qubit Hilbert space, with an eigenvector $|\psi\rangle$ such that $U|\psi\rangle = e^{2\pi i\varphi}|\psi\rangle$, and we want to find the phase, i.e. $\varphi \in [-1/2, 1/2)$.

2.1 Show that the following measurement on the top qubit



produces a result 0 with probability $p = \cos^2(\pi\varphi)$.

2.2 Explain what do you need to add to the circuit so that, no matter the result of the measurement, the final state is back to $|0\rangle \otimes |\psi\rangle$. Do not remove any gates from the circuit above, only add gates to the right of the measurement gate.

2.3 Briefly describe how this can be used to estimate the value of φ, describing any limitations.