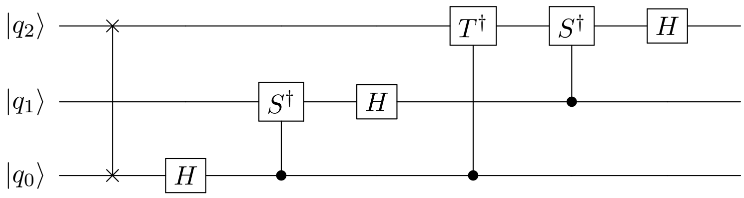
Quantum Computing Epiphany Assignment 4

Q1 U_{FT} is a unitary operator, so $U_{FT}^{-1} = U_{FT}^{\dagger}$, so

$$U_{FT}^{-1}|x\rangle = \frac{1}{2^{n/2}} \sum_{y=0}^{2^n-1} e^{-2\pi i x y/2^n} |y\rangle.$$

The circuit is given by taking the circuit for U_{FT} given in lectures and conjugating all the operators and reversing the order, giving



Q2

$$U_{FT}|x\rangle = \frac{1}{\sqrt{8}}(|0\rangle + e^{i\pi x}|1\rangle)_2(|0\rangle + e^{i\pi x/2}|1\rangle)_1(|0\rangle + e^{i\pi x/4}|1\rangle)_0 \ .$$

Applying S_0 and Z_1 gives

$$\begin{aligned} S_0 Z_1 U_{FT}|x\rangle &= \frac{1}{\sqrt{8}}(|0\rangle + e^{i\pi x}|1\rangle)_2(|0\rangle - e^{i\pi x/2}|1\rangle)_1(|0\rangle + e^{i\pi x/4}e^{i\pi/2}|1\rangle)_0 \\ &= \frac{1}{\sqrt{8}}(|0\rangle + e^{i\pi(x+2)}|1\rangle)_2(|0\rangle + e^{i\pi(x+2)/2}|1\rangle)_1(|0\rangle + e^{i\pi(x+2)/4}|1\rangle)_0 \end{aligned}$$

where for qubit 2 we note that $\exp(2\pi i) = 1$, so

$$U_{FT}^{\dagger} S_0 Z_1 U_{FT}|x\rangle = |x + 2 \pmod 8\rangle$$

with the mod 8 being due to $\exp(2\pi i) = 1$ again.

Q3 If we take $y = 3$ for example, $r = 330$. $\gcd(3^{165} - 1, 713) = 23$, which gives us $713 = 23 \times 31$.