Comparing system reliabilities with ill-known probabilities

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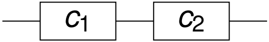
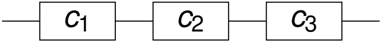
Introduction
Problem



System 1



System 2

Is system 1 more reliable than system 2 ?

- If working probabilities p_i of components c_i are precisely known → *OK*
- What happens when p_i are imprecise (lie in intervals) → ?

Introduction
System reliability

$$R^k(p_1,\ldots,p_n) = \sum_{A\subseteq\mathscr{C}^k} d_A \prod_{i\in A} p_i^{\alpha^k_{A,i}}$$

Notations :

- $\mathscr{C}^k$: set of components of system k
- $\alpha^k_{A,i}$: number of component of type i (p_i) in subset A

Hypotheses :

- Each system is coherent $\rightarrow$ R is increasing along with p_i ;
- Components working probabilities are independent ;
- p_i are expressed as intervals : $p_i \in [\underline{p_i}, \overline{p_i}]$

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Interval Comparison (IC) method

The interval $[\underline{R}, \overline{R}]$ can be obtained by

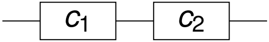
$$\underline{R} = \inf_{p_i \in [\underline{p}_i, \overline{p}_i]} R = \sum_{A \subseteq \mathscr{C}} d_A \prod_{i \in A} \underline{p}_i^{\alpha_{A,i}}$$

$$\overline{R} = \sup_{p_i \in [\underline{p}_i, \overline{p}_i]} R = \sum_{A \subseteq \mathscr{C}} d_A \prod_{i \in A} \overline{p}_i^{\alpha_{A,i}}.$$

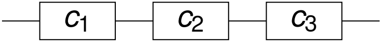
$$S^1 \succ_{IC} S^2 \text{ iff } \underline{R}^1 > \overline{R}^2$$

The calculation of IC method : simple but sometimes too rough

Numerical example with IC method



System 1

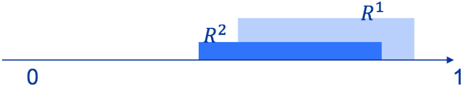


System 2

$$p_1 \in [0.7, 0.9], p_2 \in [0.8, 1] \text{ and } p_3 \in [0.8, 0.9].$$

$$R^1 = p_1 \cdot p_2 \in [0.56, 0.9]$$

$$R^2 = p_1 \cdot p_2 \cdot p_3 \in [0.448, 0.81]$$



S^1 and S^2 are incomparable according to the IC method.

Difference Comparison (DC) method

$R^{1-2} := R^1 - R^2$

R^{1-2} $>$ $=$ $<$ 0 ?

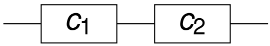
$$\underline{R}^{1-2} = \inf_{p_i \in [\underline{p_i}, \overline{p_i}]} R^1 - R^2$$

$$= \inf_{p_i \in [\underline{p_i}, \overline{p_i}]} \sum_{A \subseteq \mathscr{C}^1} d_A \prod_{i \in A} p_i^{\alpha^1_{A,i}} - \sum_{B \subseteq \mathscr{C}^2} d_B \prod_{i \in B} p_i^{\alpha^2_{B,i}}$$

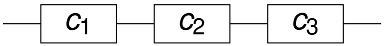
$$S^1 \succ_{DC} S^2 \text{ iff } \underline{R}^{1-2} > 0, \forall p_i \in [\underline{p_i}, \overline{p_i}]$$

With DC method, S^1 and S^2 may be comparable but R^{1-2} needs more computations.

Numerical example with DC method



System 1



System 2

$$R^1 = p_1 \cdot p_2, \qquad R^2 = p_1 \cdot p_2 \cdot p_3$$

$$R^{1-2} = p_1 \cdot p_2 \cdot (1 - p_3)$$

$$\underline{R}^{1-2} = \inf_{\substack{p_1 \in [0.7,0.9],\\ p_2 \in [0.8,1],\\ p_3 \in [0.8,0.9]}} p_1 \cdot p_2 \cdot (1 - p_3) = 0.7 \cdot 0.8 \cdot 0.1 = 0.056 > 0$$

$S^1 \succ_{DC} S^2$, but $S^1 \not\succ_{IC} S^2$!

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Proposition

Proposition 1

If $S^1 \succ_{IC} S^2$, then $S^1 \succ_{DC} S^2$.

DC is

- more precise than *IC*, and still gives guarantees
- potentially much more complex to compute

$\Rightarrow$ when does it remain easy ?

Proposition

Proposition 2

If S^1 and S^2 have distinct component types, then R^{1-2} is globally monotonic, and

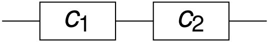
$$\underline{R}^1 - \overline{R}^2 = \underline{R}^{1-2}.$$

If j first types in S^1, $n-j$ last in S^2, then

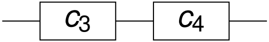
$$\underline{R}^{1-2} = R^1(\underline{p}_1, \ldots, \underline{p}_j) - R^2(\overline{p}_{j+1}, \ldots, \overline{p}_n).$$

In this particular case, *DC* and *IC* coincide.

Example



System 1



System 2

$$R^1 - R^2 = p_1 \cdot p_2 - p_3 \cdot p_4 \in [\underline{p_1} \cdot \underline{p_2} - \overline{p_3} \cdot \overline{p_4}, \overline{p_1} \cdot \overline{p_2} - \underline{p_3} \cdot \underline{p_4}]$$

Proposition

Proposition 3

If a component C_j is

- present in both systems S^1 and S^2 but
- **at most once** in each of them

then

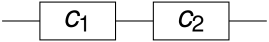
- $\underline{R}^{1-2}$ reached for $p_j \in \{\underline{p}_j, \overline{p}_j\}$

If k components $C_1, \ldots, C_k$ like that, $\underline{R}^{1-2}$ reached on vertices

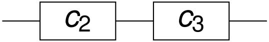
$$\times_{j=1}^{k} \{\underline{p}_j, \overline{p}_j\}$$

Exponential, but still finite set of values to check (doable if k small)

Example



System 1



System 2

$$R^1 - R^2 = p_1 \cdot p_2 - p_3 \cdot p_2 = p_2 \cdot (p_1 - p_3)$$

- if $p_1 = 0.8, p_3 = 0.7,\ p_2 \in [0.7, 0.9]$

$$\underline{R}^{1-2} = \underline{\mathbf{p}}_{\mathbf{2}} \cdot (p_1 - p_3)$$

- if $p_1 = 0.8, p_3 = 0.9,\ p_2 \in [0.7, 0.9]$

$$\underline{R}^{1-2} = \overline{\mathbf{p}}_{\mathbf{2}} \cdot (p_1 - p_3)$$

Proposition

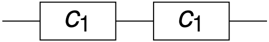
Proposition 4

If some components appear

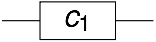
- in both systems S^1 and S^2
- more than once in at least one of them

then R^{1-2} is in general a non-monotonic polynomial, and finding $\underline{R}^{1-2}$ is a NP-hard problem.

Example



System 1



System 2

$$R^1 - R^2 = p_1^2 - p_1$$

- if $p_1 \in [0.4, 0.6]$ we have

$$\underline{R}^{1-2} = -0.25$$

obtained for $p_1 = 0.5$ (not one bound)

Contents

Conclusions and future works

Conclusions

1. Two methods (IC/DC) to get guaranteed comparisons of system reliabilities
2. IC method easy to compute, but conservative,
3. DC less conservative, but computationally complex in some cases

Perspectives

1. Using approximated bounds to get $\underline{R}^{1-2}$
2. if S^1 and S^2 incomparable, which information to make them comparable ?
3. Consider other comparison rules (E-admissibility)

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Thanks for your attention !