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ALGEBRAIC GEOMETRY III/IV

Problems, set 2.

Exercise 6. Let $C, D \subset \mathbb{P}^2$ be two curves defind by $F(X,Y,Z) = X^2 + Y^2 - 3Z^2 - 2YZ$, $G(X,Y,Z) = X^2 - 2Z^2 - 2YZ$.

(a) Calculate the resultant

$$\mathrm{Res_Z(F(X,Y,Z), G(X,Y,Z))}$$

and factorize it.

(b) Check that $[0,0,1] \notin C, D$, calculate the points of intersection and check that $[0,0,1]$ doesn't lie on any line connecting two of these points.

(c) Use (a) and (b) to determine the intersection multiplicities of the intersection points.

Exercise 7. Calculate the intersection points of the line $L : 2Y - Z = 0$ and C_F in $\mathbb{P}^2$ with $F(X,Y,Z) = XZ^2 - Y^3$ and the "old" intersection multliplicities (of a line and a curve). Then calculate $I_R(L, C_F)$ with current methods.

Exercise 8. Find the intersection points and the intersection multiplicities of $F(X,Y,Z) = 2XZ^2 + X^2Z + Y^2Z - X^3$ and $G(X,Y,Z) = 2XZ - X^2 - Y^2$.

Exercise 9. Find the intersection points and the intersection multiplicities of $F(X,Y,Z) = Z^4 + X^2Z^2 - Y^4$ and $G(X,Y,Z) = Z^6 + X^2Z^4 - Y^6$.

Exercise 10. Find the intersection points and the intersection multiplicities of $F(X,Y,Z) = X^2 + Y^2 - Z^2$ and $G(X,Y,Z) = X^2 + Y^2 - 4XZ$.

Exercise 11. Let $F(X,Y,Z) = X^3 + Y^3 - XYZ$ and

$$G(X,Y,Z) = \det \begin{pmatrix} \frac{\partial^2 F}{\partial X^2} & \frac{\partial^2 F}{\partial X \partial Y} & \frac{\partial^2 F}{\partial X \partial Z} \\ \frac{\partial^2 F}{\partial Y \partial X} & \frac{\partial^2 F}{\partial Y^2} & \frac{\partial^2 F}{\partial Y \partial Z} \\ \frac{\partial^2 F}{\partial Z \partial X} & \frac{\partial^2 F}{\partial Z \partial Y} & \frac{\partial^2 F}{\partial Z^2} \end{pmatrix}.$$

Calculate $G(X,Y,Z)$ and finde the intersection points and the intersection multiplicities of F and G. Decide for each intersection point whether it is a singular point or not.