Dr. Norbert Peyerimhoff, Durham University 6/2/2006

ALGEBRAIC GEOMETRY III/IV

Problems, set 3.

Please hand in your solutions to the exercises **2, 4, 8, 15** on **Thursday, 16 February**, in the lecture.

Exercise 12. Let $C \subset \mathbb{P}^2$ be a curve. Which of the following properties of C imply the others:

(a) C is irreducible.

(b) C is non-singular.

(c) C doesn't have multiple components.

For the cases that property A doesn't imply property B, find an explicit example of a curve having property A but not property B.

Exercise 13. Let F, G be as in Exercise 9. The result was

$$I_P(C_F, C_G) = \begin{cases} 12, & \text{if } P = [1,0,0], \\ 4, & \text{if } P = [1,0,\pm i], \\ 2, & \text{if } P = [0,1,\pm 1], \\ 0, & \text{otherwise.} \end{cases}.$$

Calculate, for all these intersection points $P \in C_F \cap C_G$, the multiplicities $m_P(C)$, $m_P(D)$ and the tangent lines and check the "Multiplicity Inequality"-Theorem for this example.

Exercise 14. Let $F(X,Y,Z) = X^3 + 3X^2Z + XY^2 - XZ^2 + Y^2Z - 3Z^3$. Show that $P_1 = [1,2,-1]$ and $P_2 = [1,-2,-1]$ are singularities of C_F. Using this information, decide whether C_F is irreducible or not.

Exercise 15.

(a) Does there exist an irreducible curve in $\mathbb{P}^2$ of degree four with exactly four singularities?

(b) Let C be a curve without multiple components and six different singularities $P_1, \ldots, P_6$ satisfying $m_{P_1}(C) = m_{P_2}(C) = 2$, $m_{P_3}(C) = m_{P_4}(C) = m_{P_5}(C) = 3$ and $m_{P_6}(C) = 4$. Give a lower bound for the degree of C.

(c) Does there exist a quartic $C \subset \mathbb{P}^2$ without multiple components with precisely one triple point and three double points? If yes, construct such a curve.