Dr. Norbert Peyerimhoff, Durham University 16/3/2006

ALGEBRAIC GEOMETRY III/IV

Information about Exercises.

All **solutions** of this course will be posted at the following web-address by Thursday, 16 March, after the lecture:

http://www.maths.dur.ac.uk/ dma0np/alggeom0506/alggeom.html

Fourth Year Exercises.

The following exercises are only for **fourth year students**. There are no solutions for these exercises, but the Puiseux expansions are given at the end of each exercise.

Exercise A. Let $C \subset \mathbb{P}^2$ be the curve defined by

$$F(X,Y,Z) = X^3 + Y^3 + 3XYZ.$$

Calculate the first 2 nonzero terms of the Puiseux series of C at $P = [0,0,1]$, i.e., find n and the first two terms of a power series $\psi(T) = \sum_{i=1}^{\infty} T^i$ such that

$$F(T^n, \psi(T), 1) = 0$$

for small enough $T \in \mathbb{C}$.

(The final answer should be: $n = 2$ and $\psi(T) = \sqrt{3}iT + \frac{1}{6}T^4 + \dots$.)

Exercise B. Let $C \subset \mathbb{P}^2$ be the curve defined by

$$F(X,Y,Z) = X^2Y^5 + 8Y^7 + 6X^2Y^4Z - X^4Z^3.$$

Find the first two nonzero terms of the Puiseux series of C at $P = [1,0,0]$, i.e., find n and the first two terms of a power series $\psi(T) = \sum_{i=1}^{\infty} T^i$ such that

$$F(1, T^n, \psi(T)) = 0$$

for small enough $T \in \mathbb{C}$.

(The final answer is: The Puiseux expansion even terminates after the second term and we have $n = 3$ and $\psi(T) = T^5 + 2T^7$, i.e.,

$$F(1, T^3, T^5 + 2T^7) = 0.$$

Exercise C. Let $C \subset \mathbb{P}^2$ be the curve defined by

$$F(X, Y, Z) = Y^4Z^3 + 2X^3Y^2Z^2 + 4X^5YZ + X^6Z + X^7.$$

Find the first two nonzero terms of the Puiseux series of C at $P = [0, 0, 1]$, i.e., find n and the first two terms of a power series $\psi(T) = \sum_{i=1}^{\infty} T^i$ such that

$$F(T^n, \psi(T), 1) = 0$$

for small enough $T \in \mathbb{C}$.

(The final answer is: The Puiseux expansion even terminates after the second term and we have $n = 4$ and $\psi(T) = iT^6 + \zeta_8 T^7$, where $\zeta_n = e^{2\pi i/n}$, i.e.,

$$F(T^4, iT^6 + \zeta_8 T^7) = 0.$$