Prof. Norbert Peyerimhoff, Durham University 12/3/2014

Algebraic Geometry III/IV

Problems, set 8. To be handed in on **Thursday, 20 March 2014**, in the lecture.

Exercise 11. Find the genus of a non-singular model of the irreducible curve $C_F \subset \mathbb{P}^2_{\mathbb{C}}$ with $F(X,Y,Z) = Y^5 - X^5 + X^2Z^3$. Follow the steps outlined in the lectures, namely:

(a) Check that $[0,1,0] \notin C_F$, so that the map $\pi : C_F \to \mathbb{P}^1_{\mathbb{C}}$, $\pi([a,b,c]) = [a,c]$ is well defined.

(b) Find the sets $R = C_F \cap C_{F_Y} \subset \mathbb{P}^2_{\mathbb{C}}$ and $B = \pi(R)$. Check that B contains at least 3 points.

(c) Find all singularities of C (they are necessarily a subset of R).

(d) Carry out the blow-up procedures to obtain a non-singular model $\widetilde{C}$.

(e) Choose a triangulation $\mathcal{T}$ of $\mathbb{P}^1_{\mathbb{C}}$ with B as its vertex set and determine the numbers V, E, F of the triangulation of $\widetilde{C}$ induced from the triangulation $\mathcal{T}$.

(f) Calculate $g(\widetilde{C})$.