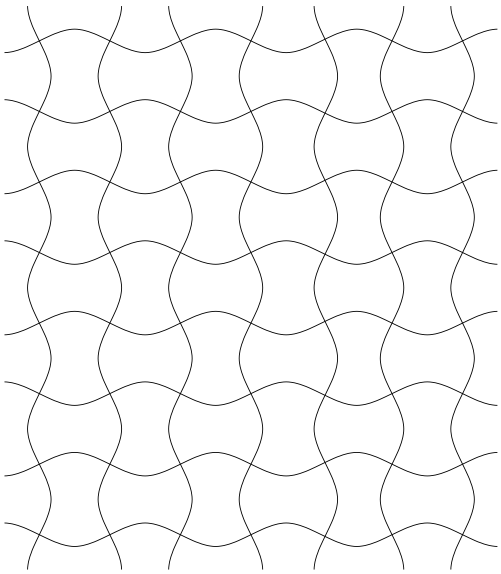
Dr. Norbert Peyerimhoff, Durham University 24/10/2008

Geometry III/IV

Problems, set 2 (Exercises 4 and 5 to be handed in on 3 November 2008).

Exercise 4. Let Γ be the symmetry group of the pattern of congruent tiles shown here:



(a) Write down all centres of rotational symmetry, together with angles of rotation, and all axes of reflectional symmetry.

(b) Determine the translation subgroup of Γ.

(c) Find a set of generators of Γ.

(d) Determine the derived group Γ'.

Exercise 5. Prove the remark after Definition 1.17: Let $\Gamma \subset I(\mathbb{R}^n)$. Γ is discrete if and only if, for every ball $B_r = \{x \in \mathbb{R}^n \mid \|x\| \leq r\}$ we have

$$|\{f \in \Gamma \mid f(0) \in B_r\}| < \infty.$$

Exercise 6. Let $\Gamma \subset I(\mathbb{R}^n)$ be a group of isometries. Show that the translation subgroup $T(\Gamma)$ is normal in Γ and that

$$\Gamma' \cong \Gamma / T(\Gamma).$$