Dr. Norbert Peyerimhoff, Durham University 7/11/2008

Geometry III/IV

Problems, set 4.

Exercise 9.

(a) Let $\pi_1, \pi_2 \subset \mathbb{R}^3$ be two planes with coordinate axes and $f : \pi_1 \to \pi_2$ be the parallel projection obtained by parallel rays orthogonal to π_1. Assume that $f(x,0) = (x,0)$, i.e., points on the x-axis of π_1 are mapped to the same points on the x-axis of π_2. Assume that the parallel rays intersect π_2 at an angle $\alpha \in (0, \pi/2]$. Calculate $f(x,y) \in \mathbb{R}^2$ and prove that f maps circles to ellipses (ellipses are given as solutions of

$$\left(\frac{x - x_0}{a}\right)^2 + \left(\frac{y - y_0}{b}\right)^2 = 1$$

with $a, b > 0$).

(b) Using (a), prove the following fact about ellipses: Let l be a chord of an ellipse. Then the midpoints of all chords of the ellipse parallel to l lie on a straight line.

Exercise 10.

(a) Find the affine transformation $f(x) = Ax + b$ which maps the points $0, e_1, e_2 \in \mathbb{R}^2$ to the points $(4,7), (1,1), (3,3) \in \mathbb{R}^2$, respectively. Calculate also f^{-1}.

(b) Find the affine transformation $g(x) = Cx + d$ which maps the points $0, e_1, e_2 \in \mathbb{R}^2$ to the points $(0,1), (2,4), (4,4) \in \mathbb{R}^2$, respectively.

(c) Using (a) and (b), calculate the affine transformation mapping the points $(4,7), (1,1), (3,3)$ to the points $(0,1), (2,4), (4,4)$ respectively.