Dr. Norbert Peyerimhoff, Durham University 8/12/2008

Geometry III/IV

Problems, set 8.

Exercise 20. Find the dual statement to Pappus' Theorem. If you do it correctly, you will end up with a statement called Brianchon's Theorem.

Exercise 21.

(a) Without using Proposition 3.16, prove that

$$[p_1, p_2; p_0, p_3] = 1 - \frac{1}{[p_0, p_1; p_2, p_3]}.$$

(b) Show that the four Points

$$p_0 = [0, 2, 1], p_1 = [1, 1, 1], p_2 = [2, 4, 3], p_3 = [1, -1, 0] \in \mathbb{R}P^2$$

lie on a Line and calculate $[p_0, p_1; p_2, p_3]$.

(c) Prove the statement in Remark 5, namely, if A, B, C, D lie on a screen $\pi \subset \mathbb{R}P^3$, then

$$[A, B; C, D] = \frac{AC}{CB} \Big/ \frac{AD}{DB}.$$

Exercise 22. Let $C \subset \mathbb{R}P^2$ be the conic defined by

$$C := \{[x] \in \mathbb{R}P^2 \mid x^\top A x = 0\} \text{ with } A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 3 \end{pmatrix}.$$

Show that C is non-empty and non-singular and calculate the pole of the Line $l = \{[x] \in \mathbb{R}P^2 \mid x_2 - 4x_3 = 0\}$ with respect to C.