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Geometry III/IV

Problems, set 9.

Exercise 23. Let the conic $C := \{[x_1, x_2, x_3] \mid x_2^2 - 2x_1x_3 = 0\}$ be given. Show that $C \subset \mathbb{R}P^2$ is non-singular and non-empty.

(a) Calculate the polar Line of the Point $P = [1, 0, 0]$.

(b) Calculate the pole of the Line $l = \{[y_1, y_2, y_3] \mid y_1 - 2y_3 = 0\}$.

(c) Find a projective transformation $f : \mathbb{R}P^2 \to \mathbb{R}P^2$ such that

$$f(C) = \{[x_1, x_2, x_3] \mid x_1^2 + x_2^2 - x_3^2 = 0\}.$$

Exercise 24. Let $A = [1, 0, 0]$, $B = [0, 1, 0]$, $C = [0, 0, 1]$, $D = [1, 1, 1] \in \mathbb{R}P^2$. Let $A' = AD \cap BC$, $B' = BD \cap CA$ and $C' = CD \cap AB$. Let $X = BC \cap B'C'$, $Y = CA \cap C'A'$ and $Z = AB \cap A'B'$.

(a) Describe explicitely the duals l_A, l_B, l_C, l_D of A, B, C, D?

(b) Show that X, Y, Z are collinear by calculating all occuring Points and Lines.

(c) Write down the dual statement of the result that you have proved in (b).

Exercise 25. There is a way to dualize conics.

(a) Given a non-singular conic

$$C = \{[x] \in \mathbb{R}P^2 \mid x^\top Ax = 0\}.$$

For every $P = [x] \in C$ let

$$l_P := \{[y] \mid x^\top Ay = 0\}$$

denote its tangent Line and $P^\top$ be the dual of l_P. Prove that $P^\top = [Ax]$ and conclude that the set

$$C^\top := \{P^\top \mid P \in C\} \subset \mathbb{R}P^2$$

is, again, a non-singular conic. $C^\top$ is called the dual conic to C.

(b) By construction any Point of a conic dualizes to a tangent of the dual conic and vice versa. Try to dualize the Theorem of Pascal. If you do it correctly, you end up with another version of *Brianchon's Theorem.*

Recall that the webpage for Geometry III/IV with the script of this term's lectures and all exercises and solutions is at

`http://www.maths.dur.ac.uk/~dma0np/geom0809/geometry.html`

MERRY CHRISTMAS !!!