

Answers to Logic Problems

Question 1

1. *True statement.* If the two odd numbers are $a = 2n + 1$ and $b = 2m + 1$ then their product is $ab = 2(2nm + n + m) + 1$ and, therefore, again odd.
2. *False statement.* We only need to find real numbers a, b, c such that $b^2 - 4ac < 0$ and $ax^2 + bx + c = 0$ does not have two real solutions. Choose, for example, $(a, b, c) = (1, 0, 1)$. Then $b^2 - 4ac = -4 < 0$ and $x^2 + 1 = 0$ has only the two purely imaginary solutions $-i, i$.
3. *True statement.* Since $\frac{1}{n}$ becomes arbitrarily small as n becomes arbitrarily large, we can find a large enough natural number n such that $\frac{1}{n} < \epsilon$. This has to do with the fact that the sequence $x_n = \frac{1}{n}$ converges to 0.
4. *True statement.* "7 is divisible by 3" is false and "9 is a prime number" is also false. Since $A \Rightarrow B$ is true for two false statements A, B , the combined statement is true.
5. *True statement.* "1001 is a prime number" is false since we have $1001 = 7 \cdot 11 \cdot 13$. " $\int_0^\pi \sin(x^2)dx < 4$ " is true since we have $\sin(x^2) \leq 1$ and, therefore,

$$\int_0^\pi \sin(x^2)dx \leq \int_0^\pi 1dx = \pi < 4.$$

Therefore, the combined statement is true.

Question 2

Because of 1, today can only be Monday or Wednesday. Let us consider the consequences of both cases.

Assume that today is Monday! Since 3 must be true, Anna must be the lecturer. By 6, Tom must be the builder. By exclusion, Max must then be the electrician. But this would imply, by 7, that today is Wednesday, which is a contradiction.

Assume that today is Wednesday! This implies that Tom or Anna is the lecturer. If Anna is the lecturer, then Tom cannot be the lecturer, but then, by 2, Max would have to be the lecturer, a contradiction to Anna being the lecturer. Therefore Anna cannot be the lecturer, so Tom must be the lecturer. Then, by 4, Anna must be the electrician and, by exclusion, Max

must be the builder. This is the only remaining possibility. One easily checks that the combination

today = Wednesday, Tom = lecturer, Anna = electrician, Max = builder makes all statements 1-7 true.

Here is an alternative, very short solution:

- Statement 2 implies that Tom or Max must be the lecturer. This implies that Anna cannot be the lecturer.
- We conclude from statement 3 that today must be Wednesday (since Anna is not the lecturer).
- We conclude from statement 5 that Tom must be the lecturer (since Anna is not the lecturer).
- We conclude from statement 4 that Anna is the electrician.
- Since Tom is the lecturer and Anna the electrician, Max must be the builder.

Of course, once this solution is found, it has to be checked that in this situation all 7 statements are true!

Question 3 Truth tables

A	B	(not A) or B	A and B	not A	(A and B) or (not A)
False	False	True	False	True	True
False	True	True	False	True	True
True	False	False	False	False	False
True	True	True	True	False	True

The truth tables show that both statements are equivalent.