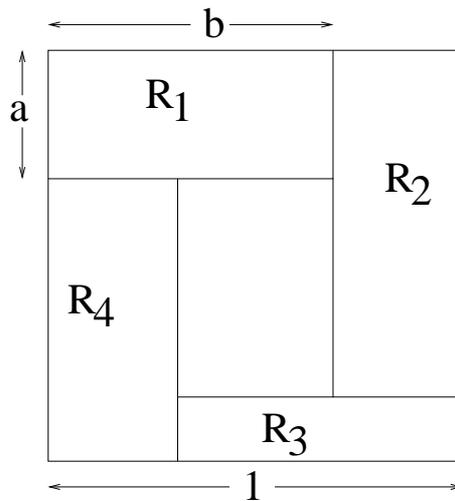


## Formulate Mathematical Conditions and Find Solutions (Week 6)

**Question 1 (Slightly modified Problem from Taylor 1989)** A square with sides of length 1 is divided up into five rectangles as shown below. Let  $a, b$  be the sides of rectangle  $R_1$ . Show the following: If all four outer rectangles  $R_1, R_2, R_3, R_4$  have the same area then  $a + b = 1$ .



**Question 2** We call a  $3 \times 3$  matrix with real entries a **magic square** if the entries of each row add up to zero and the entries of each column add up to zero and the entries along both diagonals add up to zero. Study (and answer) the following questions and justify your results using strict mathematical arguments:

- Is a magic square uniquely determined by its entries in the first row?
- Is a magic square uniquely determined by its entries in the second row?

**Question 3** Find the radius  $r > 0$  of eight equal sized circles  $C_1, C_2, \dots, C_8$  arranged around a circle  $C_0$  with radius 1 such that none of the circles  $C_i$  overlap and each of the circles  $C_i$  touches only the central circle  $C_0$  and two of the other circles  $C_1, \dots, C_8$ . For illustration, the radius  $r > 0$  in the configuration is too small, but increasing  $r$  appropriately leads to the described configuration.

