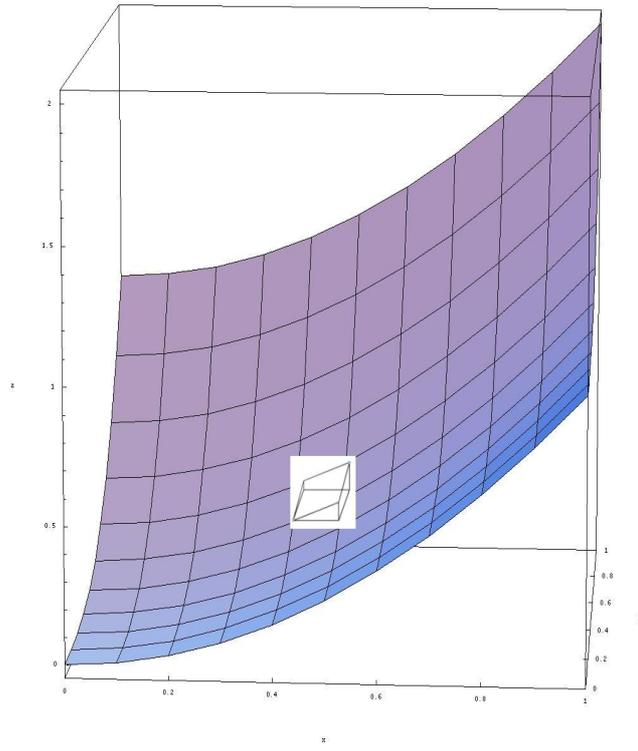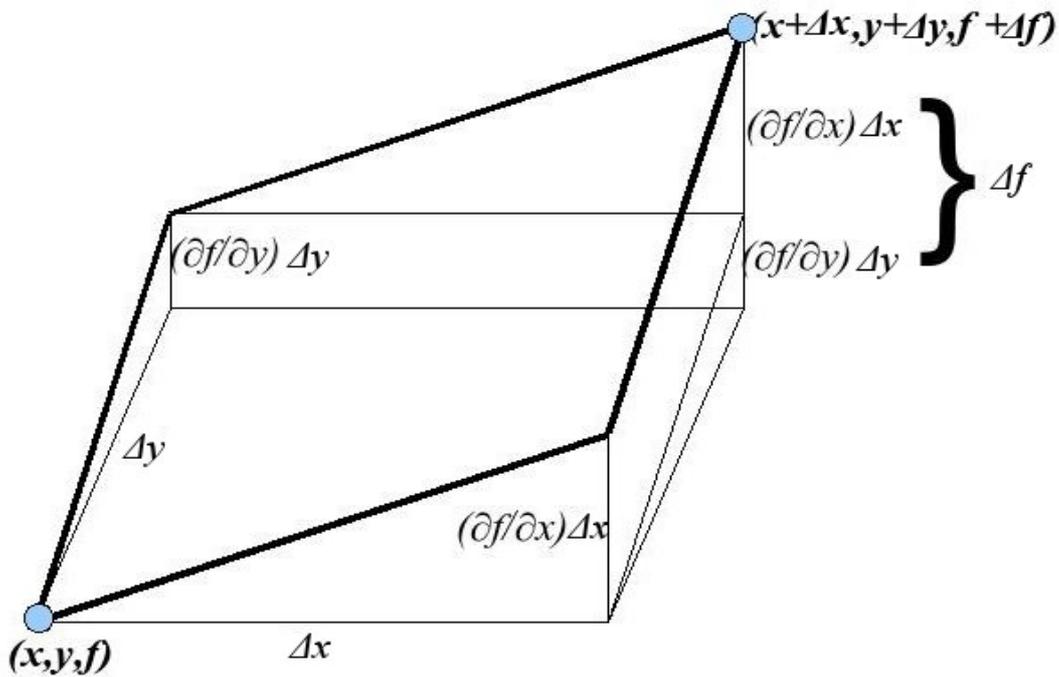For a function of two variables, the function z=f(x,y) represents a two-dimensional surface. Eg

At very small scales this becomes approximately flat. Below we magnify the small portion of the surface in white above:

Δx represents a small change in x, Δy represents a small change in y, and Δf represents the corresponding small change in the function (height) f.

We can see geometrically (see figure) that $\Delta f \approx \partial f / \partial x \, \Delta x \; + \; \partial f / \partial y \, \Delta y$.

Infinitesimally (meaning we take the limit "as small as possible") we simply send $\Delta x \to dx$, $\Delta y \to dy$, $\Delta f \to df$ and the approximation signs becomes equals signs. Ie

$$df = \partial f / \partial x \, dx \; + \; \partial f / \partial y \, dy$$