Jacobs University
School of Engineering and Science

Fall Term 2009

# ESM 1B, Homework 1

**Due Date:** 14:00 Friday, September 18, 2009.

Explain your answers! Problems marked ($\star$) are bonus ones.

**1.1.** For vectors $\vec{a} = (0, 2, 1)$ and $\vec{b} = (1, -1, 0)$, find $\vec{a} + \vec{b}$, $\vec{b} - 2\vec{a}$, $\vec{a} \cdot \vec{b}$, $\vec{a} \times \vec{b}$, $|\vec{a}|$, $|\vec{b}|$, the cosine of the angle between $\vec{a}$ and $\vec{b}$, the area of the parallelogram spanned by $\vec{a}$ and $\vec{b}$.

**1.2.** Let $\Pi$ be a parallelepiped. Consider the three vectors connecting a vertex of $\Pi$ to the centers of the three faces meeting at this vertex. Define a new parallelepiped $\Pi'$ as the parallelepiped spanned by these three vectors. Assuming that the volume of $\Pi$ is known, find the volume of $\Pi'$.

**1.3.** (a) Prove the bac-cab formula

$$\vec{a} \times (\vec{b} \times c) = \vec{b}\,(\vec{a} \cdot \vec{c}) - \vec{c}\,(\vec{a} \cdot \vec{b}).$$

*Hint:* choose appropriate coordinate system.

(b) Give an example showing that the cross-product is not associative:

$$\vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}.$$

(c) Use properties of scalar triple product to show that

$$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$$

**1.4.** ($\star$) Using vector methods, prove that the segments connecting midpoints of opposite edges in a tetrahedron intersect at one point, and that this point bisects each of these segments.

*Hint:* compute position vectors of midpoints of these segments and show that these vectors coincide.