

ESM 1B, Homework 10

Due Date: 14:00 Thursday, December 3, 2009.

Explain your answers! Problems marked (★) are bonus ones.

10.1. Solve (find general solutions of) the following differential equations by separation of variables:

$$(a) x^2 y^2 \frac{dy}{dx} + 1 = y; \quad (b) \sqrt{y^2 + 1} dx = xy dy.$$

10.2. Solve the following exact equations:

$$(a) 2xy dx + (x^2 - y^2) dy = 0; \quad (b) e^{-y} dx - (2y + xe^{-y}) dy = 0.$$

10.3. Solve the following equations by finding an integrating factor:

$$(a) y^2 dx - (xy + x^3) dy = 0; \quad (b) \left(y - \frac{1}{x}\right) dx + \frac{dy}{y} = 0.$$

10.4. Solve the following homogeneous equations:

$$(a) x^2 \frac{dy}{dx} = y(x + y); \quad (b) \left(x \sec \frac{y}{x} + y\right) dx - x dy = 0.$$

10.5. Solve the following equations:

$$(a) \frac{dy}{dx} - \frac{2y}{x} = -x^2 y^2, \quad y(1) = 1;$$

$$(b) 2xy^3 \frac{dy}{dx} + y^4 = 1, \quad y(1) = 0;$$

10.6. (★) Newton's law of cooling can be written in the form

$$\frac{dT(t)}{dt} = -k(T(t) - T_0),$$

where t is time, $T = T(t)$ is the temperature of an object, $T_0 < T$ is the temperature of the outside medium, and k is a positive constant. Using Newton's law of cooling, determine how long it takes for a cup of tea at the initial temperature of 90° to cool down to 40° . The outside temperature is 20° . Express the answer through k . Give a rough estimate for k based on your experience.