Jacobs University
School of Engineering and Science Fall Term 2009

ESM 1B, Homework 3

Due Date: 14:00 Thursday, October 1, 2009.

Explain your answers! Problems marked ($\star$) are bonus ones.

3.1. Find all first-order partial derivatives of the following functions:

$$\text{(a) } f(x,y) = x - y + \sin(xy); \qquad \text{(b) } f(x,y,z) = \tan^{-1}(yz/x).$$

For the function in (a), find all second-order partial derivatives.

Verify by explicit computation that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ in this example.

3.2. Suppose that a function $f(x,y)$ satisfies the following equation:

$$\frac{\partial^2 f}{\partial x \partial y} = 0.$$

Prove that $f(x,y) = g(x) + h(y)$, where g and h are functions of one variable.

3.3. For the following differentials, determine whether they are exact:

(a) $x\,dx + y\,dy$; (b) $x\,dy + y\,dx$; (c) $x\,dy - y\,dx$; (d) $(y\cos(xy) + 1)\,dx + x\cos(xy)\,dy$.

For the exact differentials, find the corresponding potentials.

3.4. *Cylindrical coordinates* (ρ, ϑ, ζ) are defined by the following formulas:

$$x = \rho\cos\vartheta, \quad y = \rho\sin\vartheta, \quad z = \zeta.$$

where (x,y,z) are the Cartesian coordinates. Suppose that $f(x,y,z)$ is a function, whose partial derivatives with respect to the Cartesian coordinates are known. Write a formula for the partial derivatives of f with respect to the cylindrical coordinates.

3.5. The equation

$$3y = z^3 + 3xz$$

defines z as an implicit function of x and y. Evaluate all the three second order partial derivatives of z with respect to x and y.

Verify that z is a solution of equation

$$x\frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 z}{\partial x^2} = 0$$

3.6. (a) Compute directional derivative of the function

$$f(x,y) = xe^{x/y}$$

at the point $(1,1)$ along the vector $(1,-1)$.

(b)($\star$) Consider the function

$$f(x,y) = g(x + \sin y) + h(\cos(xy)).$$

Find the directional derivative of f at the point $(\pi, \pi/2)$ along the vector $(1,1)$ in terms of the derivatives of g and h.