Jacobs University
School of Engineering and Science

Fall Term 2009

ESM 1B, Homework 4

Due Date: 14:00 Thursday, October 8, 2009.

Explain your answers! Problems marked $(\star)$ are bonus ones.

4.1. Using the chain rule, transform the equation

$$\frac{\partial^2 \varphi}{\partial x^2} = \frac{\partial^2 \varphi}{\partial y^2}$$

to new coordinates $s = \frac{1}{2}(x+y)$, $t = \frac{1}{2}(x-y)$. Show that φ has the form

$$f(x+y) + g(x-y)$$

for some functions f and g of one variable.

4.2. Suppose that a function f satisfies the equation

$$yf_x + xf_y = 0.$$

Transform this equation to new coordinates $s = x^2 - y^2$, $t = 2xy$ to show that f is a function of $x^2 - y^2$ only.

4.3. Find the second-order approximation of the following function at $(1, 2)$:

$$f(x, y) = \exp(x^2 + y^2).$$

4.4. Find all stationary points of the function

$$f(x, y) = x^3 + 2x^2 - x - y^3 + 3y^2 + 3y + 6$$

Determine the nature of the stationary points (maxima, minima, saddle points or none of these).

4.5. The temperature of a point (x, y, z) on the unit sphere $x^2 + y^2 + z^2 = 1$ is given by

$$T(x, y, z) = 1 + xy - yz.$$

Find the temperature of the hottest and the coldest points on the sphere.

4.6. Give an example of a function on $\mathbb{R}^2$ having

(a) a local minimum at $(0, 0)$ and a local maximum at $(1, 0)$;

(b)$(\star)$ a local maximum at $(0, 0)$ and local minima at $(1, 0)$, $(0, 1)$, $(-1, 0)$, $(0, -1)$;

(c)$(\star)$ saddle points at $(0, 0)$, $(1, 1)$, $(-1, -1)$, $(-1, 1)$, and $(1, -1)$.