Jacobs University School of Engineering and Science Fall Term 2009

ESM 1B, Homework 5

Due Date: 14:00 Thursday, October 22, 2009.

Explain your answers! Problems marked ($\star$) are bonus ones.

5.1. Find the stationary points of $f(x,y,z) = x^3 + y^3 + z^3$ subject to the following constraints

$$x^2 + y^2 + z^2 = 1, \quad x + y + z = 1.$$

5.2. A rectangular parallelepiped is symmetric with respect to planes $x = 0$, $y = 0$, $z = 0$, and has all eight vertices on the ellipsoid

$$x^2 + 3y^2 + 3z^2 = 1.$$

Find the maximal and the minimal surface area and volume of such a parallelepiped.

5.3. Find the area of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1.$$

Hint: reduce the problem to finding the area under the graph of some function.

5.4. Evaluate the integral of $f(x,y)$ over rectangle $0 \leq x \leq a$, $0 \leq y \leq b$ for the functions

$$\text{(a)} \quad f(x,y) = \frac{2x}{x^2 + y^2}; \qquad \text{(b)} \quad f(x,y) = (b - y + x)^{-3/2}.$$

5.5. Evaluate the integral of

$$f(x,y,z) = x^2 + y^2 + z^2$$

over the rectangular parallelepiped bounded by six planes $x = \pm a$, $y = \pm b$, $z = \pm c$.

5.6. ($\star$) Let $f(x,y) = x^2 - y$, and define $F(x,y)$ as

$$F(x,y) = \begin{cases} x^2 - y + e^{-1/x^2} \sin \frac{1}{x}, & x \neq 0, \\ x^2 - y, & x = 0 \end{cases}$$

Find extrema of $f(x,y)$ under constraints $F(x,y) = 0$.