Jacobs University
School of Engineering and Science

Fall Term 2009

Perspectives in Mathematics I, Homework 1

Due Date: Monday, September 28, in class.

Explain your answers! Problems marked ($\star$) are bonus ones.

1.1. Assume that the radius of Earth is equal to 6400km.

(a) Show that the sum of distances between any three cities in the world does not exceed 12800π km.

(b) The city A is located at the distance 1000km from the cities B and C, the trajectories of the flights from A to B and from A to C are perpendicular to each other. Estimate the distance between B and C.

1.2. Prove that any spherical triangle has an inscribed and a circumscribed circle.

1.3. ($\star$)

A *polar* of a line on a sphere is a pair of corresponding poles.

(a) Given a spherical line segment of length α, prove that the polars of all spherical lines intersecting this segment cover a set of area 4α.

(b) Given several spherical line segments of total length less than π, prove that there is exists a spherical line disjoint fron each of the segments.

1.4. (a) Find the projective transformation that takes 0, 1, ∞ to 1, 3, 0 respectively.

(b) Compute $[1, 2, 4, \infty]$.

1.5. Given a non-identity map

$$f(x) = \frac{ax + b}{cx + d}$$

show that its square $f(f(x))$ is identity if and only if $a + d = 0$.

1.6. Find a transformation of the projective plane that takes the line $x = 0$ to the line at infinity.

1.7. Let A and B be points on a circle. Does there exist a projective transformation preserving the circle and taking AB to a diameter?

1.8. Show that if a line l does not intersect a circle C, then there exists a projective transformation preserving C and taking l to the line at infinity.