Jacobs University
School of Engineering and Science

Fall Term 2009

Perspectives in Mathematics I, Homework 2

Due Date: Wednesday, October 14, in class.

Explain your answers! Problems marked $(\star)$ are bonus ones.

2.1. Show that points z_1, z_2, $z_3 \in \mathbb{C}$ are collinear (belong to one line) if and only if $\dfrac{z_1 - z_2}{z_1 - z_3} \in \mathbb{R}$.

2.2. (a) Show that points A, B, C, $D \in \mathbb{R}^2$ lie on one circle if and only if $\angle ABC + \angle ADC = \pi$.

(b) Use part (a) to show that z_1, z_2, z_3, $z_4 \in \mathbb{C}$ lie on one circle or one line if and only if $[z_1, z_2, z_3, z_4] \in \mathbb{R}$.

2.3. (a) Find a Moebius transformation taking 0, 1, and 2 to -1, 0, and ∞ respectively.

(b) Find a Moebius transformation f taking the upper half-plane $\operatorname{Im} z > 0$ to the unit disk with $f(i) = 0$.

(c) Draw the images of the sets $\{z \in \mathbb{C} \,|\, \operatorname{Re} z \in \mathbb{Z}\}$ and $\{z \in \mathbb{C} \,|\, \operatorname{Im} z \in \mathbb{Z}\}$ under the map constructed in (b).

2.4. (a) Find the infimum and supremum of the sum of angles of a polygon with n sides.

(b) Given an acute-angled polygon P and two lines l and m containing two disjoint egdes of P, show that l and m are disjoint.

2.5. (a) Show that any pair of parallel lines can be transformed to any other pair of parallel lines by an isometry.

(b) Show that all horocycles are pairwise congruent.

2.6. (a)$(\star)$ Given a point C moving along an arc AB of a circle or horocycle, prove that $\angle ABC + \angle BAC - \angle ACB$ is constant.

(b)$(\star)$ Given a convex quadrilateral $ABCD$ with equal opposite sides, show that the lines containing sides AB and CD diverge.