Jacobs University
School of Engineering and Science

Fall Term 2009

Perspectives in Mathematics I, Midterm problems

Due Date: Monday, November 2, in class.

Explain your answers! Problems marked ($\star$) are bonus ones.

M.1. (a) Solve the equation $|z|z = -1$.

(b) Find the locus $\mathcal{R}e\,((z+1)/z) = 1$.

M.2. Find a circle C such that the inversion i_C takes the unit circle to the line $z + \overline{z} = -2$.

M.3. Find any Möbius map taking the domain $\{i(\overline{z} - z) > \overline{z} + z > 0\}$ to the domain $\{z\overline{z} < 2,\ z + \overline{z} > 2\}$.

M.4. Let P_1 be a hyperbolic triangle with vertices $(\infty, e^{\frac{\pi i}{3}}, e^{\frac{2\pi i}{3}})$, and let P_2 be a triangle with vertices $(0, e^{\frac{\pi i}{3}}, \frac{1+i}{2})$. If P_1 is congruent to P_2?

M.5. Let AB and CD be congruent closed intervals on the sphere. Show that there exists a point M on the sphere, such that triangles MAB and MCD are congruent.

M.6. Show that the diameter of a circle inscribed in a triangle in $\mathbb{H}^2$ does not exceed $\log 3$.

M.7. Is it possible to tile a regular hyperbolic triangle with side of length 100 by regular triangles with side of length 1?

M.8. What is the type of a composition of reflections in consequtive sides of P, if P is

(a) an ideal triangle in $\mathbb{H}^2$?

(b)($\star$) a regular ideal quadrilateral in $\mathbb{H}^2$?

M.9. ($\star$) Let f be a linear-fractional transformation of $\mathbb{RP}^1$ such that $f(x_0) \neq x_0$ and $f(f(x_0)) = x_0$ for some $x_0 \in \mathbb{RP}^1$. Show that $f(f(x)) = x$ for every $x \in \mathbb{RP}^1$.