Jacobs University
School of Engineering and Science

Fall Term 2009

Introductory Topology, Midterm problems

Due Date: Tuesday, November 3, 1pm.

Explain your answers! Problems marked ($\star$) are bonus ones.

M.1. Show that a topological space X is Hausdorff if and only if the diagonal

$$\Delta = \{(x,x) | x \in X\} \subset X \times X$$

is closed in $X \times X$.

M.2. (a) Show that a continuous map of the closed interval onto the square cannot be bijective.

(b) Show that any two disjoint compact subsets of a Hausdorff space have disjoint neighborhoods.

M.3. Let $A \subset X$, and let $C \subset X$ be a connected subset. Assume that C meets both A and $X \setminus A$. Show that C also meets the boundary ∂A of A.

M.4. Is it true that every open subset U of X is the interior of the closure $\overline{U}$?

M.5. A *cone* over a topological space X is defined by

$$C(X) = (X \times [0,1]) / \sim$$

with equivalence relation $(x,1) \sim (y,1)$ for any $x, y \in X$. Show that

(a) $C(D^n) \approx D^{n+1}$; (b) $C(S^n) \approx D^{n+1}$.

M.6. (a) Prove that $S^n \setminus S^m \approx \mathbb{R}^{m+1} \times S^{n-m-1}$, where $S^m \subset S^n$.

Consider an embedding

$$S^m \vee S^n = (S^m \times \{y\}) \cup (\{x\} \times S^n) \subset S^m \times S^n, \quad x \in S^m,\ y \in S^n.$$

Show that

(b) $(S^1 \times S^1)/(S^1 \vee S^1) \approx S^2$;

(c)($\star$) $(S^m \times S^n)/(S^m \vee S^n) \approx S^{m+n}$.

M.7. ($\star$) Recognize the configuration space of a plane hinge mechanism $\langle 3,1,1,3;3 \rangle$ consisting on 5 consequtive rods of lengths $3,1,1,3,3$ forming a closed polygonal line, where the last one (of length 3) is fixed.