Jacobs University
School of Engineering and Science

Fall Term 2010

Discrete Mathematics, Homework 4

Due Date: Thursday, November 11, in class.

Problems marked ($\star$) are bonus ones.

4.1. Let A be the adjacency matrix for the graph G. Show that the number of triangles in G is the trace of A^3 divided by 6.

4.2. A connected graph G has even/odd number of vertices, and even/odd number of edges. For every of the four possibilities either provide an example of Eulerian graph or show it does not exist.

4.3. Show that if every edge of G belongs to an odd number of cycles then G is Eulerian.

4.4. The *line graph* $L(G)$ of a graph G is defined as follows: the vertices of $L(G)$ are the edges of G, and two vertices are joined by an edge if and only if the corresponding edges in G share a vertex.

(a) Find $L(G)$ for $G = K_n, K_{2,3}$.

(b) Let G be connected and regular (i.e., degrees of all the vertices are the same). Show that $L(G)$ is Eulerian.

4.5. Show that every Hamiltonian graph is 2-connected.

4.6. Show that if G is Eulerian then $L(G)$ is Hamiltonian.

4.7. ($\star$) Let Z_1 be the graph shown on Figure 1, and G be a 2-connected graph.

(a) Show that if G has no induced subgraphs isomorphic to Z_1 and $K_{1,3}$ (i.e., G is $\{Z_1, K_{1,3}\}$-*free*) then G is Hamiltonian.

(b) Show that the condition (a) is not necessary for 2-connected Hamiltonian graphs.

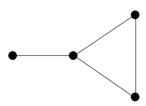

Figure 1: The graph Z_1

4.8. Draw a planar regular graph of degree 5.

4.9. Prove or disprove: if two planar graphs have F faces (regions), E edges and V vertices, then they are isomorphic.

4.10. Let G be a *maximal planar graph* of order n, i.e. G is planar, but adding any new edge leads to a nonplanar graph.

(a) Show that all the faces of G are triangles.

(b) Express the number of edges and faces of G in terms of n.

4.11. Let G be a planar connected graph on n vertices. Show that

(a) if $n < 12$ then $\delta(G) \leq 4$;

(b) if $n \geq 11$ then either G or its complement is nonplanar (a *complement* $\overline{G}$ of G is a graph on the same vertices as G, such that uv is an edge of $\overline{G}$ if and only if uv is not an edge of G).