Jacobs University
School of Engineering and Science

Fall Term 2011

ESM 1B, Homework 2

Due Date: 14:00 Wednesday, September 21.

Explain your answers! Problems marked ($\star$) are bonus ones.

2.1. Let P be a parallelepiped. Consider the three vectors connecting a vertex of P to the centers of the three faces meeting at this vertex. Define a new parallelepiped P' as the parallelepiped spanned by these three vectors. Assuming that the volume of P is known, find the volume of P'.

2.2. Find the parameter, vector and coordinate equations of the line containing points $(1, -1, 1)$ and $(2, 3, 6)$.

2.3. Find the parameter, vector and coordinate equations of the plane containing points $(1, 0, 1)$, $(0, -1, 0)$ and $(-1, 1, 1)$.

2.4. Let Π_1 be the plane containing points $A = (-3, 2, 0)$, $B = (7, 2, 0)$ and $C = (2, 3, 2)$. Plane Π_2 passes through A and is orthogonal to the line BC, whilst plane Π_3 passes through B and is orthogonal to the line AC. Find the coordinates of the point, where the three planes intersect.

2.5. Which of the following equations define a sphere?

1) $$(x-1)^2 + (y+3)^2 + z^2 + x - 23 = 0;$$

2) $$x^2 + y^2 + 3z - 5 = 0;$$

3) $$(x+1)^2 + y^2 + (z-2)^2 + 4y + 5 = 0.$$

Explain your answer.

2.6. Find the distance from point $A = (-1, 1, 0)$ to the plane containing points

$$B = (0, 0, 0), \quad C = (0, 1, 2), \quad D = (1, 0, -1).$$

Also, find the distance from B to the line CD, and the distance between lines AB and CD.

2.7. ($\star$) Find the distance between opposite edges of a regular tetrahedron with edge of length 1.