

### ESM 1B, Homework 3

**Due Date:** 14:00 Wednesday, September 28.

Explain your answers! Problems marked (★) are bonus ones.

**3.1.** Find all first-order partial derivatives of the following functions:

$$(a) f(x, y) = x - y + \cos(xy); \quad (b) f(x, y, z) = \cot^{-1}(yz/x).$$

For the function in (a), find all second-order partial derivatives.

Verify by explicit computation that  $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$  in this example.

**3.2.** Suppose that a function  $f(x, y)$  satisfies the following equation:

$$\frac{\partial^2 f}{\partial x \partial y} = 0.$$

Prove that  $f(x, y) = g(x) + h(y)$ , where  $g$  and  $h$  are functions of one variable.

**3.3.** For the following differentials, determine whether they are exact:

$$(a) x dx - y dy; \quad (b) x dy + y dx; \quad (c) x dy - y dx; \quad (d) (y \sin(xy) + 1) dx + x \sin(xy) dy.$$

For the exact differentials, find the corresponding potentials.

**3.4.** Compute the total differential of the function

$$f(x, y, z) = \cos(z + e^{xy}).$$

**3.5.** (a) Compute directional derivative of the function

$$f(x, y) = ye^{x/y}$$

at the point  $(1, 1)$  along the vector  $(1, -1)$ .

(b) Compute directional derivative of the function

$$f(x, y, z) = x^2 yz - \cos xyz$$

at the point  $(1, 1, 0)$  along the vector  $(1, 2, -1)$ .

(c)(★) Consider the function

$$f(x, y) = g(x + \sin y) + h(\cos(xy)).$$

Find the directional derivative of  $f$  at the point  $(\pi, \pi/2)$  along the vector  $(1, 1)$  in terms of the derivatives of  $g$  and  $h$ .

**3.6.** Give an example of a function  $f(x, y)$ , whose directional derivative  $f_{\vec{v}}$  at  $(0, 0)$  along  $\vec{v} = (2, 1)$  is 1 and whose directional derivative  $f_{\vec{u}}$  at the same point along  $\vec{u} = (1, -2)$  is  $-1$ .