ESM 1B, Homework 6

Due Date: 14:00 Wednesday, October 19.

Explain your answers! Problems marked ($\star$) are bonus ones.

6.1. The temperature of a point (x, y, z) on the unit sphere $x^2 + y^2 + z^2 = 1$ is given by

$$T(x, y, z) = 1 + xy - yz.$$

Find the temperature of the hottest and the coldest points on the sphere.

6.2. Find the stationary points of $f(x, y, z) = x^3 + y^3 + z^3$ subject to the following constraints

$$x^2 + y^2 + z^2 = 1, \quad x + y + z = 1.$$

6.3. Find the maximum and minimum values of $f(x, y) = x^2 + 2y^2 + 3z^2$ on the intersection of planes

$$x + y + z = 1 \qquad \text{and} \qquad x - y + 2z = 2$$

6.4. ($\star$) Let $f(x, y) = x^2 - y$, and define $F(x, y)$ as

$$F(x, y) = \begin{cases} x^2 - y + e^{-1/x^2} \sin \frac{1}{x}, & x \neq 0, \\ x^2 - y, & x = 0 \end{cases}$$

Find extrema of $f(x, y)$ under constraints $F(x, y) = 0$.