Jacobs University
School of Engineering and Science

Fall Term 2011

ESM 1B, Homework 7

Due Date: 14:00 Wednesday, October 26.

Explain your answers! Problems marked ($\star$) are bonus ones.

7.1. Find the area of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1.$$

Hint: reduce the problem to finding the area under the graph of some function.

7.2. Evaluate the integral of $f(x, y)$ over rectangle $0 \leq x \leq a$, $0 \leq y \leq b$ for the functions

$$\text{(a)} \quad f(x, y) = \frac{2y}{x^2 + y^2}; \qquad \text{(b)} \quad f(x, y) = (b - x + y)^{-3/2}.$$

7.3. Evaluate the integral of

$$f(x, y, z) = x^2 + y^2 + z^2$$

over the rectangular parallelepiped bounded by six planes $x = \pm a$, $y = \pm b$, $z = \pm c$.

7.4. Let D be a unit square $0 \leq x \leq 1$, $0 \leq y \leq 1$.

(a) Evaluate the integral of the function $f(x, y) = x \sin(x + y)$ over D.

(b)($\star$) Use the definition of integral to show that the integral of $f(x, y) = \sin^3(x + y)$ over D does not exceed 1.

7.5. Find the volume of the given solid:

(a) bounded by the cylinder $x^2 + y^2 = 1$ and the planes $x = 0$, $z = 0$, $y = z$ (in the first octant, i.e. $x, y, z \geq 0$);

(b) bounded by plane $z = 0$ and surfaces $z = 2x + y^2$, $x = y^2$, $x = y^3$.

7.6. Evaluate the integral

$$\int_0^1 dx \int_x^1 e^{x/y}\, dy$$

by determining the domain of integration and reversing the order of integration.