Jacobs University
School of Engineering and Science

Fall Term 2011

ESM 1B, Homework 9

Due Date: 14:00 Wednesday, November 9.

Explain your answers! Problems marked ($\star$) are bonus ones.

9.1. Parametrize the curve $x^4 = y^3$. Find the normal and tangent components of acceleration with respect to the chosen parameter. Find the curvature of this curve as a function of (x, y).

9.2. (a) Find the arc-length parameter on the parabola $x^2 = y$, setting the initial point (i.e. where $s = 0$) to be $(0, 0)$.

(b) Compute the length of the curve $y = x^3$, $-1 \le x \le 1$.

9.3. Give a coordinate equation of the tangent plane to the surface

$$x = \sin u, \quad y = \cos v, \quad z = u + v$$

at point (u_0, v_0).

9.4. Let $\vec{a}(t) = (\cos \pi t, \sin \pi t, 2t)$, and $\vec{b}(t) = (t, 2t, 3t + 1)$. Evaluate the integral

$$\int_0^1 \left[\vec{a} \left(\frac{d\vec{a}}{dt} \cdot \vec{b} + \vec{a} \cdot \frac{d\vec{b}}{dt} \right) + \frac{d\vec{a}}{dt} (\vec{a} \cdot \vec{b}) \right] dt$$

9.5. (a) Write an equation of the tangent line to the curve $x = y$ on the surface $x^2 - y^2 + z = 2$ at the point $(1, 1, 2)$;

(b) Write the coordinate equation of the plane tangent to the surface

$$xy + \cos z = 0$$

at the point $(1/2, -\sqrt{3}, \pi/6)$.

(c) Write the coordinate equation of the plane tangent to the surface

$$\sqrt{x} + \sqrt{y} + \sqrt{z} = \sqrt{c}$$

at the point (x_0, y_0, z_0) of the surface.

9.6. (a) Compute the surface area of the paraboloid $z = x^2 + y^2$, $0 \le z \le 1$;

($\star$) Compute the area of the part of the surface $x^2 + y^2 = Rx$ contained inside the sphere $x^2 + y^2 + z^2 = R^2$.