Jacobs University
School of Engineering and Science

Fall Term 2011

ESM 1B, Homework 1

Due Date: 14:00 Wednesday, September 14.

Explain your answers! Problems marked ($\star$) are bonus ones.

1.1. For vectors $\vec{a} = (0, 2, 1)$ and $\vec{b} = (1, -1, 0)$, find $\vec{a} + \vec{b}$, $\vec{b} - 2\vec{a}$, $\vec{a} \cdot \vec{b}$, $\vec{a} \times \vec{b}$, $|\vec{a}|$, $|\vec{b}|$, the cosine of the angle between $\vec{a}$ and $\vec{b}$, the area of the parallelogram spanned by $\vec{a}$ and $\vec{b}$.

1.2. Suppose $\vec{u}$, $\vec{v}$, $\vec{w}$ are three vectors with the property that $\vec{u}$ is orthogonal to $\vec{v} - \vec{w}$, $\vec{v}$ is orthogonal to $\vec{u} - \vec{w}$. Show that $\vec{w}$ is orthogonal to $\vec{v} - \vec{u}$.

1.3. Prove the bac-cab formula

$$\vec{a} \times (\vec{b} \times c) = \vec{b}\,(\vec{a} \cdot \vec{c}) - \vec{c}\,(\vec{a} \cdot \vec{b}).$$

Hint: choose appropriate coordinate system.

1.4. Give an example showing that the cross-product is not associative:

$$\vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}.$$

1.5. ($\star$) Using vector methods, prove that the segments connecting midpoints of opposite edges in a tetrahedron intersect at one point, and that this point bisects each of these segments.

Hint: compute position vectors of midpoints of these segments and show that these vectors coincide.