Jacobs University
School of Engineering and Science

Fall Term 2011

Linear Algebra I, Homework 9

Due Date: Friday, November 25, in class.

Problems marked ($\star$) are bonus ones.

9.1. Find the Jordan normal form and the associated basis of the following matrices:

$$(a)\ \begin{pmatrix} 4 & 6 & 0 \\ -3 & -5 & 0 \\ -3 & -6 & 1 \end{pmatrix} \qquad (b)\ \begin{pmatrix} 4 & -5 & 7 \\ 1 & -4 & 9 \\ -4 & 0 & 5 \end{pmatrix} \qquad (c)\ \begin{pmatrix} 1 & -3 & 0 & 3 \\ -2 & -6 & 0 & 13 \\ 0 & -3 & 1 & 3 \\ -1 & -4 & 0 & 8 \end{pmatrix} \qquad (d)\ \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 1 & \dots & 1 \\ 0 & 0 & 1 & \dots & 1 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}$$

9.2. Show that Jordan normal form (JNF) of matrix $A + \lambda I$ is $J + \lambda I$, where J is JNF of A.

9.3. Let A be a Jordan block $J_{\lambda,r}$.

(a) Find JNF of A^2.

(b) Compute $f(A)$, where f is a polynomial.

9.4. Let J be a JNF of matrix A. Compute JNF of (a) A^2; (b) A^{-1} (if $\det A \neq 0$).

9.5. (a) Let A be a matrix of rank 1. Show that $A^2 = cA$ for some $c \in \mathbb{C}$.

(b) Let $f^m = f^n$ for some distinct positive integers m, n. Does this imply that f is diagonalizable?

(c) Show that for every $A \in \mathrm{M}_n$ there is $C \in \mathrm{M}_n$ such that $C^{-1}AC = A^T$.

9.6. Let $A \in \mathrm{M}_2$. Consider an operator $L_A \in \mathrm{End}(\mathrm{M}_2)$, $L_A(X) = AX$. Given JNF of A, compute the JNF of L_A.

9.7. Compute

$$(a)\ \begin{pmatrix} 2 & 1 \\ 3 & 0 \end{pmatrix}^{15} \qquad (b)\ \begin{pmatrix} -1 & -1 \\ 4 & 3 \end{pmatrix}^{20}$$

9.8. Solve the equation

$$X^2 = \begin{pmatrix} 3 & 1 \\ -1 & 5 \end{pmatrix}$$

9.9. ($\star$) Let $f \in \mathrm{End}(\mathbb{C}^n)$, and suppose that there is $v \in \mathbb{C}^n$ such that span of all vectors $f^k(v)$ is the whole $\mathbb{C}^n$. Find all possible JNF of f.