Durham University
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Michaelmas 2022

Topics in Combinatorics IV, Homework 8 (Week 8)

Due date for starred problems: **Friday, December 2, 6pm**.

8.1. Show that the poset $J(P)$ of order ideals of a poset P is a distributive lattice.

8.2. Complete the proof of Lemma 4.30. Given a poset P with $|P| = n$, construct a map from the set of linear extensions of P to the set of saturated chains of $J(P)$ by taking $\varphi : P \to [n]$ to the chain $\hat{0} = \emptyset \lessdot I_1 \lessdot I_2 \lessdot \ldots \lessdot I_n = \hat{1}$, where $I_j = \varphi^{-1}([j])$. Show that this map is a bijection.

8.3. ($\star$) Let $w = 2\,6\,5\,1\,4\,8\,7\,10\,9\,3 \in S_{10}$. Apply the RSK algorithm to w to obtain SYT P and Q.

8.4. ($\star$) Let (P, Q) be SYT of shape $\lambda = (4, 2, 2, 2) \vdash 10$, where

$$P = \begin{array}{|c|c|c|c|} \hline 1 & 3 & 4 & 10 \\ \hline 2 & 5 \\ \cline{1-2} 6 & 7 \\ \cline{1-2} 8 & 9 \\ \cline{1-2} \end{array} \qquad Q = \begin{array}{|c|c|c|c|} \hline 1 & 2 & 5 & 6 \\ \hline 3 & 4 \\ \cline{1-2} 7 & 8 \\ \cline{1-2} 9 & 10 \\ \cline{1-2} \end{array}$$

Construct $w \in S_{10}$ which is taken to the pair (P, Q) by the RSK algorithm.