Jacobs University
School of Engineering and Science

Spring Term 2009

ESM 2B, Homework 1

Due Date: 14:00 Wednesday, 18 February 2009.

Explain your answers! Problems marked ($\star$) are bonus ones.

1.1. Are the following sets of vectors linearly independent?

(a) $\{(1,-1,0),\ (-1,0,1),\ (0,1,-1)\} \subset \mathbb{R}^3$;

(b) $\{(1,1,0),\ (1,0,1),\ (0,1,1)\} \subset \mathbb{R}^3$;

(c) $\{x^3+x^2,\ x^2-3x+1,\ x^3-2x^2+1\} \subset \mathbb{R}[x]$, where $\mathbb{R}[x]$ is the space of polynomials with $\mathbb{R}$-coefficients.

1.2. What is the dimension of $\operatorname{span}\{(1,2,1),\ (3,2,1),\ (-1,6,3)\} \subset \mathbb{R}^3$?

1.3. Find all values of a and b, such that $\{(1,b),(a,a)\}$ is a linearly dependent subset of $\mathbb{C}^2$.

1.4. Which of the following sets are $\mathbb{R}$-vector spaces?

(a) Polynomials with real coefficients of degree at least n;

(b) continuous functions $f(x)$ on $\mathbb{R}$ with $f(0)=0$;

(c) continuous functions $f(x)$ on $\mathbb{R}$ with $f(0)=1$;

(d) bounded functions on $\mathbb{R}$;

(e) arithmetic progressions with real entries;

(f) geometric progressions with real entries;

($\star$) the set of functions $\{f(x)=a\sin(x+c)\,|\,a,c\in\mathbb{R}\}$.

1.5. Which of the following maps $V\times V\to\mathbb{F}$ are inner products?

(a) $\mathbb{F}=\mathbb{C}$, $V=\mathbb{C}^2$, $(w,z)=w_1\overline{z_2}-w_2\overline{z_1}$;

(b) $\mathbb{F}=\mathbb{R}$, $V=\mathbb{R}^3$, $(x,y)=2x_1y_2-2x_2y_1+3x_3y_3$.

1.6. Which of the following maps from $\mathbb{R}^2$ to $\mathbb{R}^2$ are linear operators?

(a) $f(x,y)=(x+y,xy)$;

(b) $g(x,y)=(x-y;2y)$;

(c) $h(x,y)=(2x+y+1;y+x)$.

1.7. Which of the following maps are linear operators?

(a) $A:\mathbb{R}^4\to\mathbb{R}^3$, $A(x,y,z,t)=(x+y,y+z,z+t)$;

($\star$) $A:\mathbb{R}[x]\to\mathbb{R}[x]$, $(Ap)(x)=p(\alpha x^2+\beta)$, where $\alpha,\beta\in\mathbb{R}$ are fixed numbers.