

### ESM 2B, Homework 10

**Due Date:** 14:00 Wednesday, May 6.

Explain your answers! Problems marked (★) are bonus ones.

- 10.1.** In how many ways can 8 people be placed around a table if there are three who insist on sitting together?
- 10.2.** A royal family has children until it has a boy or until it has three children, whichever comes first. Assume that each child is a boy with probability  $\frac{1}{2}$ . Find the expected number of boys in this family and the expected number of girls.
- 10.3.** A bag contains 3 white and 2 red balls. 3 balls are drawn simultaneously at random from the bag. Let  $X$  be the number of white balls drawn.
- (a) Write down the probability function of  $X$ .
  - (b) Compute expectation and variance of  $X$ .

- 10.4.** Let  $X_1, X_2, \dots$  be independent discrete random variables taking values in  $\{-\frac{1}{2}, \frac{1}{2}\}$  with probability function

$$p(-\frac{1}{2}) = q, \quad p(\frac{1}{2}) = (1 - q),$$

and  $p(x) = 0$  for all other values of  $x$ .

- (a) Find the expectation for the random variable

$$Z_N = \frac{X_1 + \dots + X_N}{N}$$

as  $N \rightarrow \infty$ .

- (b) Let  $Y_n = 2^{-n} X_n$ . Find the expectation for the random variable

$$Z_N = Y_1 + \dots + Y_N$$

as  $N \rightarrow \infty$ .

- 10.5.** (★) A lighthouse is situated at a distance  $L$  from a straight coastline, opposite to a point  $O$ , and sends out a narrow continuous beam of light simultaneously in opposite directions. The beam rotates with a constant angular velocity. If the random variable  $Y$  is the distance along the coastline, measured from  $O$ , of the spot that the light beam illuminates, find its probability density function.