Jacobs University
School of Engineering and Science

Spring Term 2009

ESM 2B, Homework 3

Due Date: 14:00 Wednesday, March 4.

Explain your answers! Problems marked ($\star$) are bonus ones.

3.1. Find all the solutions x of the following linear systems

$$(a)\ \begin{cases} x_1 + x_2 + 2x_3 = 1 \\ 2x_1 - x_2 = 3 \\ 2x_1 + x_3 = 2 \end{cases} \qquad (b)\ \begin{cases} x_1 - x_2 + 2x_3 = 3 \\ + 2x_2 - x_3 = 10 \\ 3x_1 + 2x_2 + 5x_3 = -2 \end{cases}$$

$$(c)\ \begin{cases} 2x_1 - x_2 + x_3 = 3 \\ -x_1 + 2x_2 - x_3 = 0 \\ x_1 + x_2 = 2 \end{cases} \qquad (d)\ \begin{cases} 2x_1 - x_2 + x_3 = 5 \\ -x_1 + 2x_2 - x_3 = -4 \\ x_1 + x_2 = 1 \end{cases}$$

by identifying the coefficient matrix A, and then using Gauss elimination.

3.2. Compute the rank of the following matrices

$$(a)\ \begin{pmatrix} -1 & 1 & 1 \\ 1 & 6 & 1 \\ 4 & 2 & 0 \end{pmatrix} \quad (b)\ \begin{pmatrix} 0 & 1 & 1 \\ 3 & 2 & 3 \\ 2 & 1 & 2 \end{pmatrix} \quad (c)\ \begin{pmatrix} 1 & 0 & 1 & 0 \\ 3 & 2 & 1 & 1 \\ 4 & 0 & 2 & -1 \end{pmatrix} \quad (d)\ \begin{pmatrix} 2 & 0 & 3 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

3.3. Are the following matrices invertible? If yes, compute the inverses.

$$(a)\ \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 2 & 0 \end{pmatrix} \quad (b)\ \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix} \quad (c)\ \begin{pmatrix} 1 & 0 & 1 \\ 3 & 2 & 1 \\ 4 & 0 & 2 \end{pmatrix} \quad (d)\ \begin{pmatrix} 2 & 1 & 3 & 0 \\ 0 & 1 & -2 & 1 \\ 3 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

3.4. Compute determinants of the following matrices:

$$(a)\ \begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 3 & 2 & -1 \end{pmatrix} \quad (b)\ \begin{pmatrix} 0 & 1 & 1 \\ 3 & 2 & -3 \\ -2 & 2 & 2 \end{pmatrix} \quad (c)\ \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 4 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & 0 & 3 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$$

3.5. (a) Show that if A is an invertible matrix, then A^n is also invertible for any positive integer n.

(b) Is it true that if A^n is invertible for some positive integer n then A is invertible?

($\star$) Let A, B be matrices, and suppose that $AB = I$. Is it true that $BA = I$?

3.6. ($\star$) Let $A, B \in M_n$, and let $AB + I$ be invertible. Show that $BA + I$ is also invertible.