Jacobs University
School of Engineering and Science

Spring Term 2009

ESM 2B, Homework 4

Due Date: 14:00 Wednesday, March 11.

Explain your answers! Problems marked ($\star$) are bonus ones.

4.1. Compute the characteristic polynomial and find the eigenvalues (with multiplicities), eigenvectors, and bases of eigenspaces for the following matrices. Which of them are diagonalizable?

$$(a)\ \begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix} \qquad (b)\ \begin{pmatrix} 0 & 2 & 1 \\ -1 & 4 & 0 \\ -1 & 1 & 3 \end{pmatrix} \qquad (c)\ \begin{pmatrix} 2 & 3 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix}$$

$$(d)\ \begin{pmatrix} 0 & 1 & 1 \\ -4 & -4 & 0 \\ 0 & 1 & 3 \end{pmatrix} \qquad (e)\ \begin{pmatrix} -4 & 3 & -3 \\ 2 & -2 & 2 \\ 8 & -7 & 7 \end{pmatrix}$$

4.2. Show that

(a) the product of eigenvalues of A equals $\det(A)$;

($\star$) the sum of eigenvalues of A equals $\mathrm{tr}(A)$.

4.3. Decide which of the following matrices are diagonalizable. If so, find a diagonal matrix D and an invertible matrix S such that $A = SDS^{-1}$.

$$(a)\ A = \begin{pmatrix} 5 & 3 \\ 1 & 0 \end{pmatrix} \qquad (b)\ A = \begin{pmatrix} 0 & 0 \\ 4 & 0 \end{pmatrix} \qquad (c)\ A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad (d)\ A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 0 \\ -2 & 0 & 2 \end{pmatrix}$$

4.4. ($\star$) Let $A, B \in M_n$, and let $AB = BA$. Suppose also that $Av = \lambda v$ for some $v \neq 0$ (i.e., v is an eigenvector of A with eigenvalue λ), and $\dim \mathrm{E}_\lambda = 1$. Show that v is an eigenvector of B.